Homework Five

Precalculus: Functions, Geometry, Trigonometry, & Modelling
UCR Math-005-E01, Summer 2020

Please upload your responses to these prompts to Gradescope before 8am next Monday morning. That way I have time to read over them to prepare for the coming week. You will upload all three of these sections under a single submission on Gradescope.

gradescope.com/courses/125651

Reflection

Do you think you've understood the math for this homework? Please respond to this in detail; I want to get a complete picture of how you're doing just by reading this reflection.

Is there anything you'd like me to look at more carefully in your Exercise and Writing responses? If there is, please indicate where it is in the rest of your submission. Like, circle it with a bright color or something.

Writing

1. (RESEARCH REPORT) Read up on how Eratosthenes estimated the circumference of the earth, and on other ways humans have computed the dimensions of the earth since Eratosthenes. Write a report detailing this history. Think of either me or Jonathan as your audience for this report. Remember though that Jonathan and I

are skeptical mathematicians, so be sure to include the mathematical details of this history in your report, and try to anticipate and answer any questions we'd have as skeptical mathematicians.

2. (RESEARCH & PEDAGOGY) Familiarize yourself with the idea of a skyhook, especially a *rotating skyhook*.

   en.wikipedia.org/wiki/Skyhook_(structure)

   Then I want you to write-up how you would explain the concept of a skyhook and a rotating skyhook to an excited middle school student. The middle school student can be a sci-fi nerd if you think this helps. Be sure to highlight the advantages and disadvantages of a rotating skyhook versus a normal skyhook. Are there any compelling reasons we can't use the moon as the mount/counterweight for a skyhook?

Exercises

1. (UW) From the University of Washington's Math120 book, page 178 (pdf page 198), work through the exercises:

   13.1 13.2 13.3 13.4 13.5 13.7 13.8

2. (EVEN AND ODD FUNCTIONS) Recall that a function f is even if $f(-x) = f(x)$, and that a function f is odd if $f(-x) = -f(x)$. For example, the sine function is odd since $\sin(-x) = -\sin(x)$, while the cosine function is even since $\cos(-x) = \cos(x)$.

   (a) How can you tell if a function is even or odd from its graph?

(b) Below are a bunch of functions. Some are even functions. Some are odd functions. Identify which are even and which are odd and which are both and which are neither.

$f(x) = x^2$	$f(x) = \sin(x)$	$f(x) = \sin^2(x)$
$f(x) = \sin(x^2)$	$f(x) = x^2 \sin(x)$	$f(x) = x^2 + \sin(x)$
$f(x) = x^3$	$f(x) = 1$	$f(x) = x$
$f(x) = \sqrt{x}$	$f(x) = \sqrt{1 - x^2}$	$f(x) = e^x$
$f(x) = \ln(x)$	$f(x) = \lvert x \rvert$	$f(x) = \frac{1}{x}$

(c) Suppose that f and g are both even functions. Can you say if fg must be even or odd? What can you say about fg if f and g are both odd? What can you say about fg if f is even and g is odd?

(d) Like in the last part, investigate the even-ness or odd-ness of the function $f + g$ in the three cases: both f and g are even, both f and g are odd, and one of f and g is even and the other is odd.

3. (UW) From the University of Washington's Math120 book, page 205 (pdf page 225), work through the exercises:

15.2 15.3 15.5 15.5 15.7 15.8

4. (UW) From the University of Washington's Math120 book, page 218 (pdf page 238), work through the exercises:

16.1 16.2 16.3 16.5 16.7 16.8

5. (GEOMETRY) Let's do some miscellaneous geometry.

   (a) Write down the equation of a line that passes through the points $(1,2)$ and $(5,8)$ in the (x,y)-plane. Let's call this line ℓ

   (b) Recall that two lines are *perpendicular* to each other if they intersect and the angle formed by their intersection is a right angle. In the (x,y)-plane, if two line are perpendicular, then their slopes will be the negative reciprocal of each other. For example any line perpendicular to a line of slope $\frac{5}{2}$ will have slope $-\frac{2}{5}$. Knowing this, write down an equation of the line $\mathfrak{l}$ that passes through the point $(2,7)$ and is perpendicular to ℓ from the previous question.

   (c) Find the coordinates of the point p where ℓ and $\mathfrak{l}$ intersect.

   (d) Now write down an argument that p is the point on ℓ that is closest to the point $(2,7)$. That is, argue that among all the points on ℓ the distance between $(2,7)$ and any of those points is larger than the distance between $(2,7)$ and p.

   (e) Now reflect on the purpose of these previous tasks. What is Mike trying to teach you by asking you to do these?

   (f) (CHALLENGE) Now a new geometric setup. Let a and b be two points on a circle in the plane, the circle having center O. Let m be the midpoint of the line-segment $\overline{ab}$ (this is notation for the line-segment with endpoints a and b). Write down an argument that the line segment $\overline{Om}$ and the line segment $\overline{ab}$ are perpendicular.

(g) (CHALLENGE) Now a new geometric setup. Suppose that r and s and t are three points in the plane that do not lie on a common line. FACT: there is a unique circle in the plane that passes through these three points r and s and t. How do you find the center of this circle? HINT: thinking of the previous two tasks will help you with this.

6. (RECREATION) You're the director at an airport on the equator, and each plane at your airport can hold exactly enough fuel to fly half way around the world. The planes can also meet up in midair to share fuel. Your task is to use as few planes as possible to successfully fly one plane all the way around the world, but all planes that you use must safely land back at your airport. How many planes do you need?