Homework Six

Precalculus: Functions, Geometry, Trigonometry, & Modelling

UCR Math-005-E01, Summer 2020

Please upload your responses to these prompts to Gradescope before 8am next Monday morning. That way I have time to read over them to prepare for the coming week. Also, I think I haven't emphasized enough that the Exercises are really just for you; I only look at them if you direct me to in your Reflection. There are (terse) solutions to some exercises at the end of this document.

`gradescope.com/courses/125651`

Reflection

Do you think you've understood the math for this homework? Are you satisfied with your responses to the Writing prompts? Please respond to this in detail. I hope to get a complete picture of how you're doing just by reading this reflection.

Is there anything from the Exercises that you struggled with, Or that you still don't understand?

Writing

1. (Research Report) Imagine I'm a very busy but very curious entrepreneur, and I've hired you to do some research and report back to me.

Suppose we were to dig a tunnel *straight* from Riverside to Los Angeles. How long would this tunnel have to be? Would this tunnel breach the Earth's mantle? What's the furthest major city we could reach with a straight tunnel that *doesn't* go through the Earth's mantle? What's the furthest major city we could reach with a straight tunnel that *doesn't* go through the Earth's outer core?

Are any of these tunnels reasonable to build? Like, are they practical? What uses/advantages would these tunnels have versus just traveling over the surface of the Earth, or in the air? Do any of these tunnels have unavoidable issues that would make them useless?

In your report be sure to explain your methods to me; I'm very curious about how math and science work.

Exercises

1. (UW) From the University of Washington's Math120 book, page 234 (pdf page 254), work through the exercises:

   17.1 17.3 17.4 17.6 17.7 17.8 17.9 17.11

2. What is the equation for a line in the (x, y)-plane passing through the point $(3, 4)$ and making an angle of $\pi/3$ radians with the x-axis? What about for a line passing through the point (π, e) that makes an angle of $\pi/5$ with the y-axis?

3. (RESEARCH) What is a spherical coordinate system? Go familiarize yourself with the specific spherical coordinate system using

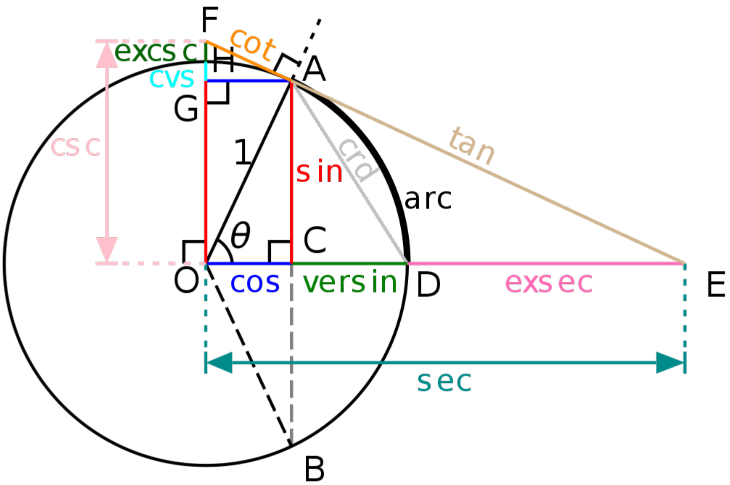
latitude and longitude that we use to describe positions on the surface of the Earth.

4. How fast are you currently moving relative to the center of the earth? Note that this isn't immediately easy because, even knowing the rate of rotation of the earth, you're not standing on the equator. (But maybe you are? With remote learning you really could be anywhere.)

5. (WEISBART) You are lying on the ground looking up at the top of the UCR bell tower. Your line of sight makes an angle of $20°$ with the ground, and the top of the tower is $1000\,\text{ft}$ away from you. How tall is the tower?

6. (WEISBART) There is a building in front of you. The angle of elevation from your position to the top of the building is $21°$. You walk $200\,\text{ft}$ towards the building and measure the angle of elevation now to be $43°$. How tall is the building?

7. (GEOMETRY) Below is a diagram of with the literal lengths associated to the unit circle correspond to the various trigonometric functions.



Even the super antiquated trig functions are included. Look at the segment labelled tan. This is why we refer to this trig function as the tangent function: $\tan(\theta)$ is the length of the line segment *tangent* to the unit circle between the point on the circle corresponding to θ and the x-axis. But we're more familiar with the definition of $\tan(\theta)$ being the slope of the hypotenuse of the triangle containing θ, or $\sin(\theta)/\cos(\theta)$. Can you prove that these definitions of $\tan(\theta)$ coincide? That is, can you prove that the length of that segment labelled tan really is equal to $\sin(\theta)/\cos(\theta)$?

8. (UW) From the University of Washington's Math120 book, page 250 (pdf page 270), work through the exercises 18.1 and 18.2.

9. What are the domain and range of the sine and cosine function? Remembering the definitions

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad \csc(x) = \frac{1}{\sin(x)} \qquad \sec(x) = \frac{1}{\cos(x)}$$

what are the domain and range of each of the tangent, secant, and cosecant functions? Which of these functions have a unique inverse?

10. Suppose that $\cos(\theta) = \frac{1}{3}\sqrt{5}$ and θ is in the first (upper right) quadrant. Without using a calculator, answer the following questions.

    (a) What is the value of $\cos(2\pi + \theta)$?

    (b) What is the value of $\cos(\pi + \theta)$?

    (c) What is the value of $\sec(\theta)$?

    (d) What is the value of $\cos(-\theta)$?

    (e) What is the value of $\cos(\pi - \theta)$?

    (f) What is the value of $\sin(\theta)$?

    (g) What is the value of $\sin(-\theta)$?

    (h) What is the value of $\csc(\theta)$?

    (i) What is the value of $\tan(\theta)$?

11. Given a function f with domain $[-3,2) \cup (2,\infty)$, a subset of the real numbers, what is the domain of each of these functions?

$$f(x+5) \qquad f(5x) \qquad 5f(x) \qquad f(x)+5 \qquad (\star)$$

    Suppose that f is a periodic function on $(2,\infty)$ with period 7. What is the period of each of the functions in ($\star$) on their corresponding domains?

12. (RECREATIONAL) Fifty natural numbers are written in such a way so that sum of any four consecutive numbers is 53. The first number is 3, the 19th number is eight times the 13th number, and the 28th number is five times the 37th number. Find the 44th number.

Hints & Solutions

There are solutions to the University of Washington's book's exercises on page 291. Here are solutions/hints to some other exercises.

2. They're $(x-3)\sqrt{3} = y-4$ and $(x-\pi)\tan\left(\frac{3\pi}{10}\right) = y-\mathrm{e}$, but you may get different lines depending on how you interpret the phrase "makes an angle with".

4. Unless you're on the equator, the answer is *not* 1000 mph. A hint: it would be zero if you were doing your homework at either the north or south pole.

5. Draw a picture. It's height is $1000\sin(20°)$.

6. Draw a picture. It's height is

$$200\frac{\tan(43°)\tan(21°)}{\tan(43°)-\tan(21°)}.$$

7. Hint: Notice that the triangles $\triangle OAC$ and $\triangle OEA$ are similar.

9. The argument to the sine and cosine functions are angles, and they return the y and x coordinate respectively on the unit circle that is associated to that angle. There is no angle you *can't* plug into sine or cosine, and every real number is an angle, so the domain of sine and cosine are all real numbers. Then the range of

sine and cosine, since they are the Cartesian coordinates of points on the *unit* circle, must be $[-1,1]$.

Looking at the formula that defines cosecant, we can't divide by zero, so the domain of cosecant will be all real numbers x except the x where $\sin(x) = 0$. Thinking to the definition of sine, these x will be $\{\ldots -2\pi, -\pi, 0, \pi, 2\pi, \ldots\}$. So the domain of cosecant consists of all real numbers besides the multiples of π. Similarly, looking at the formula that define tangent and secant, their domains cannot contain any numbers x such that $\cos(x) = 0$. So the domains of tangent and secant are all real numbers except for $\{\ldots -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots\}$.

Since tangent returns the slope of a line corresponding to an input angle, and there's an input angle for any slope you could desire, the range of tangent will be all real numbers. The range of secant and cosecant, looking at their definitions, will be the set of all reciprocals of numbers in $[-1,1]$, which will be $(-\infty,-1] \cup [1,\infty)$.

None of these functions have a unique inverse. You'd have to restrict their domains to one rotation of the circle, like the angles in $[0,2\pi)$ or something, to be able to define an inverse function.

10. (a) $\frac{1}{3}\sqrt{5}$

    (b) $-\frac{1}{3}\sqrt{5}$

    (c) $\frac{3}{\sqrt{5}}$

    (d) $\frac{1}{3}\sqrt{5}$

    (e) $-\frac{1}{3}\sqrt{5}$

(f) $\frac{2}{3}$

(g) $-\frac{2}{3}$

(h) $\frac{3}{2}$

(i) $\frac{2}{\sqrt{5}}$

12. Feel free to talk to Mike about this one ;)