MATH 144, HANDOUT 5:
RELATIONS

Recall the following definition:

Definition 1. Let A and B be sets. A **relation** R from the set A to the set B is a subset of $A \times B$. That is, R is a collection of ordered pairs (a, b) where $a \in A$ and $b \in B$.

Now consider the next definition:

Definition 2 (Domain/range of a relation)**.** If R is a relation from a set A to a set B, then:

- The **domain** of R, denoted $\mathrm{dom}(R)$, is the subset of A consisting of all the first coordinates of the ordered pairs in R.
- The **raange** of R, denoted $\mathrm{range}(R)$, is the subset of B consisting of all the second coordinates of the ordered pairs in R.

Using set-builder notation., these are:

$$\mathrm{dom}(R) = \{u \in A | (u, v) \in R \textbf{ for at least one } v \in B\}$$
$$\mathrm{range}(R) = \{v \in B | (u, v) \in R \textbf{ for at least one } u \in A\}$$

We'll also be interested in the **power set** of a set:

Definition 3. Let S be any set. The **power set** $\mathcal{P}(S)$ is the set of all subsets of S.

Example 4. If $A = \{1, 2, 3\}$, then what is $\mathcal{P}(A)$? And what is $\mathcal{P}(\emptyset)$?

Exercise 5. Determine the domain and range of each of the following relations on $\mathbb{R}$ and sketch the graph of each:

(1) $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} | x^2 + y^2 = 10\}$

(2) $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} | y^2 = x + 10\}$

(3) $T = \{(x, y) \in \mathbb{R} \times \mathbb{R} | |x| + |y| = 10\}$

(4) $Q = \{(x, y) \in \mathbb{R} \times \mathbb{R} | x^2 = y^2\}$

Exercise 6. Let S be a finite set with n elements. Prove that $\mathcal{P}(S)$ has 2^n elements.

Exercise 7. Let U be a nonempty set and R be the "subset relation" on $\mathcal{P}(U)$. That is,

$$R = \{(S, T) \in \mathcal{P}(U) \times \mathcal{P}(U) | S \subset T\}$$

(1) What is the domain of the subset relation R?
(2) What is the range of the subset relation R?
(3) Is it true that for every $S \in \mathcal{P}(U)$, there is a unique T so that $(S, T) \in R$?