MATH 144, HANDOUT 2:
SOME PROBLEMS ABOUT INDEXED COLLECTIONS OF SETS

Exercise 1. A special chessboard is 2 squares wide and n squares long. Using n dominoes that are 1 square by 2 squares, there are many ways to completely cover this chessboard with no overlap.

Determine a formula (in terms of n) which gives the total possible number of ways. Then prove your formula is correct.

Pairwise disjointness. Often when we consider infinite intersections of the form $\bigcap_{\alpha \in \Delta} A_\alpha$, we end up getting the emptyset. This can happen even when any finite subcollection of our sets have nonempty common intersection. We are sometimes also interested in the opposite setting: infinite unions over collections where any finite subcollection have empty intersection.

Definition 2. A collection of sets $\{A_\alpha\}_{\alpha \in \Delta}$ is **pairwise disjoint** if $A_\alpha \cap A_\beta = \emptyset$ for $\alpha \neq \beta$.

Exercise 3. Find examples for each of the following situation:

(1) Draw a Venn diagram of a collection of 3 sets that are pairwise disjoint.
(2) Provide an example of a collection of three sets, say $\{A_1, A_2, A_3\}$, such that the collection is *not* pairwise disjoint, but $\bigcap_{n=1}^{3} A_n = \emptyset$.
(3) Provide an example of an infinite collection of sets $\{A_\alpha\}_{\alpha \in \Delta}$, such that any finite subcollection has nonempty common intersection, but $\bigcap_{\alpha \in \Delta} A_\alpha = \emptyset$.

The following two theorems are useful generalizations of our previous results.

Theorem 4 (Generalized Distribution of Union and Intersection)**.** Let $\{A_\alpha\}_{\alpha\in\Delta}$ be a collection of sets and let B be any set. Then

(a) $B \cup \left(\bigcap_{\alpha\in\Delta} A_\alpha \right) = \bigcap_{\alpha\in\Delta} (B \cup A_\alpha)$,

(b) $B \cap \left(\bigcup_{\alpha\in\Delta} A_\alpha \right) = \bigcup_{\alpha\in\Delta} (B \cap A_\alpha)$.

Theorem 5 (Generalized DeMorgan's Law)**.** Let $\{A_\alpha\}_{\alpha\in\Delta}$ be a collection of sets. Then

(a) $\left(\bigcup_{\alpha\in\Delta} A_\alpha \right)^c = \bigcap_{\alpha\in\Delta} A_\alpha^c$,

(b) $\left(\bigcap_{\alpha\in\Delta} A_\alpha \right)^c = \bigcup_{\alpha\in\Delta} A_\alpha^c$.

Exercise 6. Prove part (a) of both of the above theorems.