MATH 144, HANDOUT 1:
NEGATION AND CONTRADICTION

Exercise 1. Negate each of the following. Disregard the truth value and the universal set.

(a) $(\forall x)(x > 3)$
(b) $(\exists x)(x \text{ is prime} \wedge x \text{ is even})$
(c) All cars are red.
(d) All fish live in water.
(e) For all $x \in \mathbb{N}$, $x^2 + x + 41$ is prime.
(f) There exists $x \in \mathbb{Z}$ such that $1/x \notin \mathbb{Z}$.
(g) There is no function f such that if f is continuous, then f is not differentiable.

Theorem 2. Prove the following statement:

For all real numbers a and b, if $a > 0$ and $b > 0$, then $\frac{2}{a} + \frac{2}{b} \neq \frac{4}{a+b}$.

Problem 3. Suppose x and y are odd integers. Prove that there does not exist an integer z so that $x^2 + y^2 = z^2$.

Consider the following theorem:

Theorem 4 (Archimedean Property)**.** For every real number $x \in \mathbb{R}$, there is a natural number $n \in \mathbb{N}$ with $n > x$.

Use the above theorem to prove:

Theorem 5. For any real number $\epsilon > 0$, there exists a natural number $N \in \mathbb{N}$ so that $\frac{1}{N} < \epsilon$.