

Given Open book decomp of M^3

Binding C , pages P , fiber bundle is

$$P \rightarrow X \rightarrow S^1$$

$$\partial X \rightarrow S^1$$

$$\cong C \times S^1$$

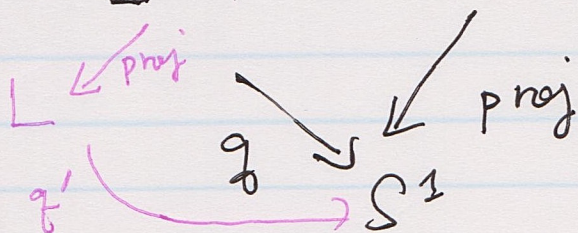
$$(M \cong C \times D^2 \cup X)$$

Also given a link L in M^3 s.t. L is transverse to the pages of X .

$$L \cap C = \emptyset \\ (so L \subseteq \text{Int } X)$$

General L has a fiberwise tubular nbhd.

$$L \times D^2 \subseteq \text{Int } X$$



q is a submersion.

q' is too. Note that $L =$ union of circles and q' on each component is a covering space proj.

Follow Thurston - Winkelnkemper

On D^2 we have the 1-form

$$\left. \begin{array}{l} u = r \cos \theta \\ v = r \sin \theta \end{array} \right\} \frac{1}{2} (u dv - v du) = \frac{1}{2} r^2 d\theta$$

CART. POLAR

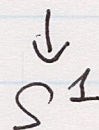
we get $dudv = r dr d\theta$, the standard volume form on D^2 .

In TW's notation, $X = W(h)$.

Let $W_0 = W - \text{Int}(L \times D^2)$, so $W_0 = W$ with

some disks removed. $\text{Def } X_0 = X - \text{Int}(L \times D^2)$,

then, as bundles, we have $X = X_0 \cup_{L \times S^1} L \times D^2$.



One crucial step in T-W is the construction of a 1-form on X with certain properties.

We need to refine this construction so that the restriction of this 1-form to $L \times D^2 \subseteq X$ will be the pullback of $\frac{1}{2}(u dv - v du)$.

This reduces to finding a suitable 1-form on X_0 whose restriction to a collar $L \times S^1 \times [1, \epsilon)$

is given by $\frac{1}{2}(u dv - v du)$. In order to

avoid conflicts with T-W's notation we should (shall) replace θ by χ henceforth.

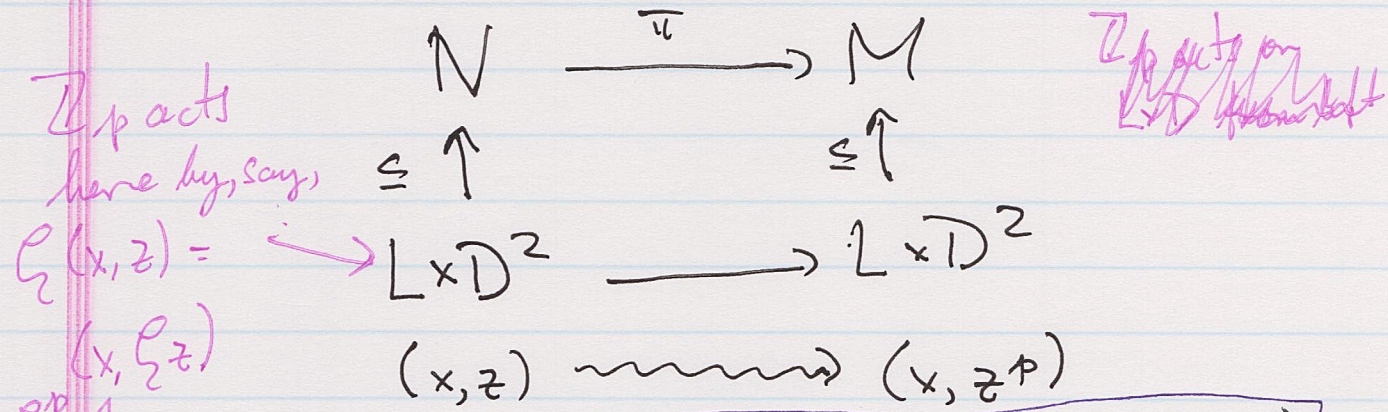
[Note: the form on $L \times D^2$ and $L \times S^1 \times [1, \epsilon)$ does not involve the L -variable!]

so it's a pullback of a form on \mathbb{R}^2

One can now construct the form α over X_0 , such that its restriction to $\partial X_0 = \partial X \sqcup L \times S^1$ has the standard forms, as in the 3rd paragraph on p. 346 of T-W. Furthermore, we can proceed as in the remainder of the paper to construct a contact form ω on M s.t. its restriction to a neighborhood of $L \times \mathbb{D}^2$ ~~is~~ is given by $\alpha + K g'^* d\phi$, where ϕ ~~is~~ given as in T-W. and K are

Now suppose we have a regular cyclic branched covering $\pi: N \rightarrow M$ of degree $p > 1$ with branch set equal to L ; to simplify the discussion, assume that p is prime (the case of a general orbit space projection requires more elaborate notation). Composing with an automorphism of \mathbb{Z}_p if necessary,

we can arrange things so that



where $\xi^p = 1$

commutes. POLAR: $(r, \psi) \rightsquigarrow (r^p, p\psi)$
FORM

Since $\pi | N - L \times \{0\}$ is a smooth submersion, if ω is a contact form on M then $\pi^* \omega$ will be a contact form on $N - L \times \{0\}$. Thus it remains to find a contact form on $L \times \mathbb{D}^2$ which extends $\pi^* \omega$ near $L \times S^1$.

An obvious first step is to write out $\pi^* \omega | L \times \mathbb{D}^2$ explicitly:

$$\pi^* \omega | L \times \mathbb{D}^2 = \frac{p}{2} r^p d\psi + K \boxed{g^{p-1} d\phi}$$

($p > 1$ prime)

~~Note that this form is 0 on $L \times \{0\}$ and hence is not a contact form on~~

Note $\pi^*\omega$ is a contact form near $\pi^{-1}[c]$, so we don't need to modify the form near $\pi^{-1}[c]$.

Note that $\pi^*\omega \wedge d\pi^*\omega = 0$ on $L \times \{0\}$ and hence $\pi^*\omega$ is not a contact form. We need to modify it so that the restriction to a neighborhood neighborhood of $L \times \{0\}$ looks like $\frac{1}{2}r^2 d\psi + k d\phi_1$, which is a \mathbb{Z}_p -invariant contact form on this nbhd.

More precisely, we need a function $F(r)$

such that $F'(r) > 0$ if $r > 0$

The (or one) construction for F has been outlined.

$$F(r) = \frac{1}{2}r^2 \quad \text{on near } 0 \leq r < \epsilon$$

Some $\epsilon > 0$ small

Note that this is \mathbb{Z}_p -invariant

$$F(r) = \frac{p}{2}r^p \quad \text{on near } 1-\epsilon < r$$

where \mathbb{Z}_p acts on $L \times D^2$ via 2 coord.

Given F , let $\mu = F(r) d\psi + k d\phi_1$.

$\mu =$ standard near $L \times \{0\}$

Then $d\mu = k \cdot F'(r) dr \wedge d\psi$ and

$$\mu \wedge d\mu = k \cdot F'(r) dr \wedge d\psi \wedge d\phi_1 \Rightarrow \mu \text{ is again a}$$

contact form on $L \times D^2 - \{0\}$ with $\mu = \pi^*\omega$ near $L \times S^1$. Since μ is standard near $L \times \{0\}$, μ is also a contact form near $L \times \{0\}$, and we are done. \square

Reminders

Given (\mathbb{Z}_p, N) , we take $M = N/\mathbb{Z}_p$,
and we ~~take~~ ^{denote} M/\mathbb{Z}_p and its image in M by L ;

L is a link in M . If we take an open book decomposition of M , then we can isotop it so that L is disjoint from the binding and ~~isotope~~ transverse to the pages

(One isotops L w.r.t. the given binding and uses the isotopy to construct a perturbed open book structure on M).