ADDITIONAL EXERCISES FOR MATHEMATICS 10B

FALL 2006

The exercises are organized by sections of the course text (Marsden and Tromba, *Vector Calculus*, Fifth Edition). Note that there are no additional exercises for Sections 7.3, 7.5 and 8.1–8.4.

Exercises for Marsden and Tromba, Section 5.1

A1. Evaluate the following iterated integral:

$$\int_0^1 \int_0^2 (x+y)\, dy\, dx$$

A2. Evaluate the following iterated integral, and describe the region R for which the expression represents a double integral over R:

$$\int_0^2 \int_0^1 (1+2x+2y)\, dy\, dx$$

Exercises for Marsden and Tromba, Section 5.2

A1. Use double integrals to find the volume of the solid defined by the inequalities $0 \leq x \leq 4$, $0 \leq y \leq 2$, $0 \leq z \leq 6-2y$.

A2. Is the following equation true or false? Give reasons to support your answer.

$$\int_{-1}^1 \int_{-1}^1 \cos(x^2+y^2)\, dx\, dy \quad = \quad 4 \cdot \int_0^1 \int_0^1 \cos(x^2+y^2)\, dx\, dy$$

Exercises for Marsden and Tromba, Section 5.3

A1. Evaluate the following iterated integral:

$$\int_1^2 \int_1^x \frac{1}{(x+y)^2}\, dy\, dx$$

A2. Evaluate the following iterated integral:

$$\int_0^1 \int_0^x \sqrt{1-x^2}\, dy\, dx$$

A3. Evaluate the following iterated integral:

$$\int_0^1 \int_0^{\sqrt{1-y^2}} (x+y)\, dy\, dx$$

A4. Find the volume of the solid defined by the inequalities $0 \leq x \leq y$, $0 \leq y \leq 1$, $0 \leq z \leq 1 - xy$.

A5. Find the volume of the solid bounded by the planes with equations $x = 0$, $y = 0$, $z = 0$ and $2x + 3y + 4z = 12$. [*Hint:* The intersection of this plane with the $xy-$plane is the line joining the points $(6, 0, 0)$ and $(0, 4, 0)$.]

Exercises for Marsden and Tromba, Section 5.4

A1. Determine the limits of integration for the double integral

$$\int \int_A f(x, y)\, dx\, dy$$

if A is the region bounded by the graphs of $x = -y$ and $x = 2y - y^2$, for both orders of integration. Find the area of A by computing this integral when $f = 1$:

A2. Interchange the order of integration in the following iterated integral:

$$\int_0^1 \int_0^x \sqrt{1-x^2}\, dy\, dx$$

A3. Interchange the order of integration in the following iterated integral:

$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \log_e(1 + x^2y^2)\, dx\, dy$$

A4. Interchange the order of integration in the following iterated integral:

$$\int_0^1 \int_{y^3}^{\sqrt{y}} \tan(xy)\, dx\, dy$$

A5. Evaluate the following iterated integral by interchanging the order of integration:

$$\int_0^1 \int_y^1 \sin x^2\, dx\, dy$$

Exercises for Marsden and Tromba, Section 6.1

A1. Let $(x, y) = \mathbf{T}(u, v) = (3u + 2v,\, 3v)$, let R be the set of points in the $xy-$plane that are on or inside the triangle with vertices $(0, 0)$, $(3, 0)$ and $(2, 3)$, and let S be the region in the $uv-$plane which maps to R under $\mathbf{T}$. Describe the region S by (a) drawing a sketch in the $uv-$plane, (b) expressing this in words or algebraic inequalities.

A2. Let $(x, y) = \mathbf{T}(u, v) = (4u + v,\, u + 2v)$, let R be the set of points in the $xy-$plane that are on or inside the parallelogram with vertices $(0, 0)$, $(1, 2)$, $(4, 1)$ and $(5, 3)$, and let S be the region in the $uv-$plane which maps to R under $\mathbf{T}$. Describe the region S by (a) drawing a sketch in the $uv-$plane, (b) expressing this in words or algebraic inequalities.

Exercises for Marsden and Tromba, Section 6.2

A1. Evaluate the following integral by changing from rectangular to polar coordinates:

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} y\,dx\,dy$$

A2. Evaluate the following integral by changing from rectangular to polar coordinates:

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} xy\,dy\,dx$$

A3. Evaluate the following integral by changing from rectangular to cylindrical coordinates:

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^4 x\,dz\,dy\,dx$$

A4. Evaluate the following integral by changing from rectangular to spherical coordinates:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \sqrt{x^2+y^2+z^2}\,dz\,dy\,dx$$

A5. Let $(x, y) = \mathbf{T}(u, v) = \left(\frac{1}{2}(u + v),\, \frac{1}{2}(u - v)\right)$, and let R be the set of points in the $xy-$plane that are on or inside the square with vertices $(1, 0)$, $(0, 1)$, $(-1, 0)$ and $(0, -1)$. Evaluate the following integral using the change of variables associated to $\mathbf{T}$:

$$\int\int_R 4(x^2 + y^2)\,dx\,dy$$

A6. Using the same definitions and procedure as in the previous exercise, evaluate the following integral:

$$\int\int_R 48xy\,dx\,dy$$

A7. Find the Jacobian
$$\frac{\partial(x,y,x)}{\partial(u,v,w)}$$
for the change of variables associated to the following transformation:
$$(x,y,z) \quad = \quad \mathbf{T}(u,v,w) \quad = \quad \big(u(1-v),\, uv(1-w),\, uvw\big)$$

A8. Find the Jacobian
$$\frac{\partial(x,y,x)}{\partial(u,v,w)}$$
for the change of variables associated to the following transformation:
$$(x,y,z) \quad = \quad \mathbf{T}(u,v,w) \quad = \quad \big(4u-v,\, 4v-w,\, u+w\big)$$

Exercises for Marsden and Tromba, Section 6.3

A1. Using cylindrical coordinates, find the centroid of a right circular cone for which the base has radius r and the height is equal to h.

A2. Using spherical coordinates, find the centroid of a hemispherical solid of radius r with uniform density.

A3. Using spherical coordinates, find the centroid of the solid of uniform density lying between two concentric hemispheres with radii r and R, where $r < R$.

Exercises for Marsden and Tromba, Section 7.1

A1. Fill this in.

A2. Fill this in.

Exercises for Marsden and Tromba, Section 7.2

A1. Fill this in.

A2. Fill this in.

Exercises for Marsden and Tromba, Section 7.4

A1. Fill this in.

A2. Fill this in.

Exercises for Marsden and Tromba, Section 7.6

A1. Fill this in.

A2. Fill this in.