ADDITIONAL EXERCISES FOR MATHEMATICS 10B

FALL 2006

The exercises are organized by sections of the course text (Marsden and Tromba, *Vector Calculus*, Fifth Edition). Note that there are no additional exercises for Sections 7.3, 7.5 and 8.1–8.4.

Exercises for Marsden and Tromba, Section 5.1

A1. Evaluate the following iterated integral:

$$\int_0^1 \int_0^2 (x+y)\, dy\, dx$$

A2. Evaluate the following iterated integral, and describe the region R for which the expression represents a double integral over R:

$$\int_0^2 \int_0^1 (1+2x+2y)\, dy\, dx$$

Exercises for Marsden and Tromba, Section 5.2

A1. Use double integrals to find the volume of the solid defined by the inequalities $0 \le x \le 4$, $0 \le y \le 2$, $0 \le z \le 6-2y$.

A2. Is the following equation true or false? Give reasons to support your answer.

$$\int_{-1}^1 \int_{-1}^1 \cos(x^2+y^2)\, dx\, dy \quad = \quad 4 \cdot \int_0^1 \int_0^1 \cos(x^2+y^2)\, dx\, dy$$

Exercises for Marsden and Tromba, Section 5.3

A1. Evaluate the following iterated integral:

$$\int_1^2 \int_1^x \frac{1}{(x+y)^2}\, dy\, dx$$

A2. Evaluate the following iterated integral:

$$\int_0^1 \int_0^x \sqrt{1-x^2}\, dy\, dx$$

A3. Evaluate the following iterated integral:

$$\int_0^1 \int_0^{\sqrt{1-y^2}} (x+y)\, dy\, dx$$

A4. Find the volume of the solid defined by the inequalities $0 \le x \le y$, $0 \le y \le 1$, $0 \le z \le 1 - xy$.

A5. Find the volume of the solid bounded by the planes with equations $x = 0$, $y = 0$, $z = 0$ and $2x + 3y + 4z = 12$. [*Hint:* The intersection of this plane with the $xy-$plane is the line joining the points $(6, 0, 0)$ and $(0, 4, 0)$.]

Exercises for Marsden and Tromba, Section 5.4

A1. Determine the limits of integration for the double integral

$$\int\int_A f(x,y)\, dx\, dy$$

if A is the region bounded by the graphs of $x = -y$ and $x = 2y - y^2$, for both orders of integration. Find the area of A by computing this integral when $f = 1$:

A2. Interchange the order of integration in the following iterated integral:

$$\int_0^1 \int_0^x \sqrt{1-x^2}\, dy\, dx$$

A3. Interchange the order of integration in the following iterated integral:

$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \log_e(1 + x^2y^2)\, dx\, dy$$

A4. Interchange the order of integration in the following iterated integral:

$$\int_0^1 \int_{y^3}^{\sqrt{y}} \tan(xy)\, dx\, dy$$

A5. Evaluate the following iterated integral by interchanging the order of integration:

$$\int_0^1 \int_y^1 \sin x^2\, dx\, dy$$

Exercises for Marsden and Tromba, Section 6.1

A1. Let $(x, y) = \mathbf{T}(u, v) = (3u + 2v,\ 3v)$, let R be the set of points in the $xy-$plane that are on or inside the triangle with vertices $(0, 0)$, $(3, 0)$ and $(2, 3)$, and let S be the region in the $uv-$plane which maps to R under $\mathbf{T}$. Describe the region S by (a) drawing a sketch in the $uv-$plane, (b) expressing this in words or algebraic inequalities.

A2. Let $(x, y) = \mathbf{T}(u, v) = (4u + v,\ u + 2v)$, let R be the set of points in the $xy-$plane that are on or inside the parallelogram with vertices $(0, 0)$, $(1, 2)$, $(4, 1)$ and $(5, 3)$, and let S be the region in the $uv-$plane which maps to R under $\mathbf{T}$. Describe the region S by (a) drawing a sketch in the $uv-$plane, (b) expressing this in words or algebraic inequalities.

Exercises for Marsden and Tromba, Section 6.2

A1. Evaluate the following integral by changing from rectangular to polar coordinates:

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} y\, dx\, dy$$

A2. Evaluate the following integral by changing from rectangular to polar coordinates:

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} xy\, dy\, dx$$

A3. Evaluate the following integral by changing from rectangular to cylindrical coordinates:

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^{4} x\, dz\, dy\, dx$$

A4. Evaluate the following integral by changing from rectangular to spherical coordinates:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx$$

A5. Let $(x, y) = \mathbf{T}(u, v) = \left(\frac{1}{2}(u + v),\ \frac{1}{2}(u - v)\right)$, and let R be the set of points in the $xy-$plane that are on or inside the square with vertices $(1, 0)$, $(0, 1)$, $(-1, 0)$ and $(0, -1)$. Evaluate the following integral using the change of variables associated to $\mathbf{T}$:

$$\int\int_R 4(x^2 + y^2)\, dx\, dy$$

A6. Using the same definitions and procedure as in the previous exercise, evaluate the following integral:

$$\int\int_R 48xy\, dx\, dy$$

A7. Find the Jacobian

$$\frac{\partial(x, y, x)}{\partial(u, v, w)}$$

for the change of variables associated to the following transformation:

$$(x, y, z) \quad = \quad \mathbf{T}(u, v, w) \quad = \quad \big(u(1-v),\ uv(1-w),\ uvw \big)$$

A8. Find the Jacobian

$$\frac{\partial(x, y, x)}{\partial(u, v, w)}$$

for the change of variables associated to the following transformation:

$$(x, y, z) \quad = \quad \mathbf{T}(u, v, w) \quad = \quad \big(4u - v,\ 4v - w,\ u + w \big)$$

Exercises for Marsden and Tromba, Section 6.3

A1. Using cylindrical coordinates, find the centroid of a right circular cone for which the base has radius r and the height is equal to h.

A2. Using spherical coordinates, find the centroid of a hemispherical solid of radius r with uniform density.

A3. Using spherical coordinates, find the centroid of the solid of uniform density lying between two concentric hemispheres with radii r and R, where $r < R$.

Exercises for Marsden and Tromba, Section 7.1

A1. Evaluate the given line integral over the indicated curve:

$$\int_C (x - y)\, ds \qquad C:\ \mathbf{r}(t) \quad = \quad (4t,\ 3t), \qquad 0 \le t \le 2$$

A2. Evaluate the given line integral over the indicated curve:

$$\int_C 4xy\, ds \qquad C:\ \mathbf{r}(t) \quad = \quad (t,\ 1-t), \qquad 0 \le t \le 1$$

Exercises for Marsden and Tromba, Section 7.2

In each of exercises A1 and A2 below, evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{r}(t)$ is the chosen parametrization for the curve C:

A1. $\mathbf{F}(x,y) = (xy,\, y), \qquad \mathbf{r}(t) = (4t,\, t), \quad 0 \le t \le 1.$

A2. $\mathbf{F}(x,y) = (3x,\, 4y), \qquad \mathbf{r}(t) = (2\cos t,\, 2\sin t), \quad 0 \le t \le \frac{\pi}{2}.$

A3. Evaluate the line integral

$$\int_C x\,dx \;+\; y\,dy \;+\; 5z\,dz$$

where C is parametrized by $\mathbf{r}(t) = (2\cos t,\, 2\sin t,\, t)$, where $0 \le t \le 2\pi$.

Exercises for Marsden and Tromba, Section 7.4

A1. Find the surface area for the portion of the graph of $z = 10 + 2x - 3y$ over the square with vertices $(0,0)$, $(2,0)$, $(0,2)$, and $(2,2)$.

A2. Find the surface area for the portion of the graph of $z = 4 + x^2 - y^2$ over the disk defined by $x^2 + y^2 \le 1$.

A3. Find the surface area for the portion of the graph of $z = xy$ over the disk defined by $x^2 + y^2 \le 16$.

Exercises for Marsden and Tromba, Section 7.6

In each of exercises A1–A3 below, evaluate the surface integral $\int\int_{\mathbf{S}} \mathbf{F} \cdot d\mathbf{S}$, for the given choices of the vector field (= vector valued function) $\mathbf{F}$ and the oriented surface S:

A1. $\mathbf{F}(x,y,z) = (3z, 4, y)$ and S is the portion of the plane $x+y+z = 1$ in the first octant.

A2. $\mathbf{F}(x,y,z) = (x,y,z)$ and S is the portion of the paraboloid $z = 9 - x^2 - y^2$ lying in the upper half-space defined by $z \ge 0$.

A3. $\mathbf{F}(x,y,z) = (x,y,z)$ and S is the portion of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant.