

General Remarks on

Stable Homotopy Theory, Part V:

Duality and Thom Spectra

by J. Michael Boardman

The material posted here is the fifth part of notes by J. M. Boardman that were mimeographed at the University of Warwick in 1965–1966 and covered the material in his 1964 D. Phil. thesis, *On Stable Homotopy Theory and Some Applications*, written at Cambridge under C. T. C. Wall.

Parts I – IV of Boardman’s notes contain his breakthrough work on constructing a mathematically sound version of stable homotopy theory which overcame the main objections to earlier attempts. Excellent summaries of this material, its impact, and more recent developments in the area are discussed in the proceedings of the 1998 conference held in honor of Professor Boardman’s sixtieth birthday [3].

In addition to an introduction with simply titled “Stable Homotopy Theory” and Parts I – V which we have already mentioned, there was also a Part VI, which was superseded by a later article by Boardman on his solution to the Conner-Floyd **Five Halves Problem** for smooth involutions on closed manifolds [2]. On the other hand, even though the material in Part V has been used and cited repeatedly in the literature and has been extremely influential (compare [1]), no detailed treatment of the topics in Part V has appeared. Therefore I am posting the original notes in order to make the contents more widely accessible.

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REFERENCES

- [1] J. C. Becker and D. H. Gottlieb. *A history of duality in algebraic topology*. Chapter 25 in “History of Topology” (I. M. James, ed.), pp. 725–745. North-Holland, Amsterdam, NL, 1999. — Modified version available online:
<http://www.math.purdue.edu/~gottlieb/Papers/duality11.pdf>
- [2] J. M. Boardman, *Cobordism of involutions revisited*. “Proceedings of the Second Conference on Compact Transformation Groups (Univ. Massachusetts, Amherst, MA, 1971), Part I,” [Springer] Lecture Notes in Math. Vol. 298, pp. 131–151, Springer-Verlag, Berlin-etc., 1972.
- [3] J. - P. Meyer, J. Morava and W. S. Wilson. “Homotopy invariant algebraic structures — A conference in honor of J. Michael Boardman. (Proc. AMS Special Session on Homotopy Theory, Baltimore, 1998),” Contemp. Math. Vol. 239. American Mathematical Society, Providence, RI, 1999.

There is a table of contents for Boardman’s notes on the next page.

Table of Contents

1.	Thom spectra.....	2
2.	Combinatorial Poincaré duality	10
3.	The Thom construction.....	17
4.	Thom isomorphisms	20
5.	Bordism and cobordism theories.....	26
6.	Transfer homomorphisms	34
7.	Riemann-Roch theorems	50
8.	Characteristic cobordism classes.....	53
9.	Some geometric homomorphisms	61
	References	67