

Mathematics 205C: Topology — I

Course Notes, Fall 2008

Detailed Table of Contents

I	Foundational material	xxx
	Coverage of topics in this unit.....	xxx
	I.1 : Basic set theory.....	xxx
	The Axiom of Choice.....	xxx
	I.2 : Products, relations and functions.....	xxx
	Definitions of relations and functions.....	xxx
	Comments on terminology.....	xxx
	Images and inverse images of subsets.....	xxx
	Disjoint unions.....	xxx
	I.3 : Cardinal numbers.....	xxx
	Partial ordering of cardinalities.....	xxx
	Cardinal arithmetic.....	xxx
	I.4 : The real number system.....	xxx
II	Metric and topological spaces	xxx
	II.1 : Metrics and topologies.....	xxx
	The basic definitions and a few examples.....	xxx
	Open sets.....	xxx
	Topological spaces.....	xxx
	Comparing and constructing topologies on a set.....	xxx
	Basic open subsets for a topology.....	xxx
	Subspace topologies.....	xxx
	Neighborhoods.....	xxx
	II.2 : Closed sets and limit points.....	xxx
	Limit points and limits of sequences.....	xxx
	Closures and interiors of subsets.....	xxx
	Convergence in general topological spaces.....	xxx
	II.3 : Continuous functions.....	xxx
	Homeomorphisms and other special mappings.....	xxx
	Different metrics determining the same topology.....	xxx
	Metric spaces of functions.....	xxx
	Piecing together continuous functions.....	xxx
	II.4 : Cartesian products.....	xxx
	Topological structures on finite products.....	xxx
	General properties of (finite) product topologies.....	xxx
	Products and morphisms.....	xxx
	Products and metric spaces.....	xxx
	Products and the Hausdorff Separation Property.....	xxx
	Infinite products.....	xxx

III	Spaces with special properties	xxx
	III.1 : Compact spaces – I.....	xxx
	The Finite Intersection Property	xxx
	Compactness and continuous mappings.....	xxx
	Products and compactness	xxx
	Compactness and infinite products	xxx
	Compact metric spaces	xxx
	III.2 : Complete metric spaces.....	xxx
	Properties of complete metric spaces.....	xxx
	Function spaces	xxx
	Intersections of nested closed sets.....	xxx
	Completions of metric spaces	xxx
	III.3 : Implications of completeness.....	xxx
	Nowhere dense and meager subspaces.....	xxx
	Baire spaces.....	xxx
	The Contraction Lemma	xxx
	III.4 : Connected spaces.....	xxx
	Connectedness and the real line	xxx
	Finding connected (sub)sets.....	xxx
	Products and connectedness.....	xxx
	Distinguishing homeomorphism types.....	xxx
	III.5 : Variants of connectedness.....	xxx
	Locally connected spaces	xxx
	Path or arcwise connectedness.....	xxx
	Analogies with connectedness.....	xxx
	Open subsets of Euclidean spaces.....	xxx
IV	Function spaces	xxx
	IV.1 : General properties.....	xxx
	Formal properties of function spaces.....	xxx
	The compact-open topology.....	xxx
	Compactness in function spaces	xxx
	IV.2 : The adjoint property	xxx
V	Constructions on spaces	xxx
	V.I : Quotient spaces	xxx
	Quotients and morphisms	xxx
	Recognizing quotient maps	xxx
	Important examples.....	xxx
	V.2 : Sums and cutting and pasting	xxx
	Disjoint union topologies	xxx
	Copy, cut and paste constructions	xxx
	Disjoint unions of families of sets	xxx

VI	Spaces with additional properties	xxx
	VI.1 : Second countable spaces	xxx
	Implications of second countability	xxx
	The reverse implications for metric spaces	xxx
	Examples involving nonmetrizable spaces	xxx
	VI.2 : Compact spaces – II	xxx
	Sequential compactness for metric spaces	xxx
	Wallace’s Theorem	xxx
	VI.3 : Separation axioms	xxx
	The T_i conditions	xxx
	Compactness and separation axioms	xxx
	References for examples	xxx
	Non-Hausdorff topologies	xxx
	VI.4 : Local compactness and compactifications	xxx
	Compactifications of noncompact spaces	xxx
	Examples of compactifications	xxx
	The Alexandroff one point compactification	xxx
	One point compactifications of Euclidean spaces	xxx
	Locally connected compactifications	xxx
	VI.5 : Metrization theorems	xxx
	Constructions for continuous functions	xxx
	Piecewise metrizable spaces	xxx
	An alternate approach	xxx
	General metrization theorems	xxx

Appendix A : Topological groups	xxx
Properties of topological groups	xxx
Analysis on topological groups	xxx
Remarks on Lie groups	xxx
Appendix B : Stereographic projection and inverse geometry	xxx
Relating stereographic projections and inversions	xxx
The conformal property for inversions	xxx
Appendix C : Homogeneity of open sets in Euclidean spaces	xxx
A local construction	xxx
Local homogeneity	xxx
A global homogeneity theorem	xxx
Some related examples	xxx
Appendix D : Normal forms for orthogonal transformations	xxx
Small dimensional orthogonal transformations	xxx
Complexification	xxx
Derivation of normal form	xxx
References	xxx
Further references	xxx
Summary of Files in the Course Directory	xxx