Math 133
Winter 2009

MORE EXERCISES FOR SECTIONS II.3 AND II.4

C1. Suppose that A, B, C are noncollinear points in a plane P, ald let $X \in \Delta ABC$. Prove that if X is a vertex of ΔABC then there are (at least) two lines L and M such that X lies on both and each contains at least three distinct points of ΔABC, but if X is not a vertex then there is only one line L in P such that $X \in L$ and L contains at least three points of ΔABC. [*Hint:* The conclusion in Exercise II.2.8 is useful for establishing part of this result.]

C2. Suppose that we are given ΔABC and ΔDEF in a plane P such that $\Delta ABC = \Delta DEF$. Prove that $\{A, B, C\} = \{D, E, F\}$. [*Hint:* Use the preceding exercise.]

C3. Suppose that we are given a triangle ΔABC in a plane P, and suppose that L is a line in P such that L contains a point X in the interior of ΔABC. Prove that L and ΔABC have (at least) two points in common.

C4. [*In this exercise we shall view points of* $\mathbb{R}^n$ *as* $n \times 1$ *column vectors and identify scalars with* 1×1 *matrices in the obvious fashion.*] Let T be an affine transformation of $\mathbb{R}^3$ and write it as $T(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$, where A is an invertible 3×3 matrix and $\mathbf{b}$ is some vector in $\mathbb{R}^3$. Let P be the plane defined by the equation $C\mathbf{x} = d$, where C is a 3×1 matrix and $d \in \mathbb{R}$, and let Q be the image of P; in other words, Q is the set of all vectors $\mathbf{y}$ such that $\mathbf{y} = T(\mathbf{x})$ for some $\mathbf{x} \in P$. Prove that Q is also a plane, and give an explicit equation of the form $U\mathbf{y} = v$ (where U is 1×3 and v is a scalar) which defines Q. [*Hint:* Solve $T(\mathbf{x}) = \mathbf{y}$ for $\mathbf{x}$ in terms of $\mathbf{y}$, A and $\mathbf{b}$.]