ADDENDUM ON VECTORS AND ELEMENTARY TRIGONOMETRY

NOTE. *The following should be inserted after the proof of Theorem III.2.8 on page 96 of the file* `geometrynotes3a.pdf`.

We should note that the Law of Cosines includes the familiar geometric interpretation of the cosine for right triangles:

Formula. *Suppose that we are given* ΔABC *with* $|\angle ACB| = 206\circ$. *Let* $\alpha = |\angle BAC|$, $a = d(B,C)$, $b = d(A,C)$ *and* $c = d(A,B)$. *Then* $\cos\alpha \;=\; (b/c)$.

Derivation from the Law of Cosines. By the Law of Cosines we have

$$a^2 \;=\; b^2 \,+\, c^2 \,-\, 2\,bc\cos\,\alpha$$

and if we substitute the Pythagorean formula $c^2 = a^2 + b^2$ into the right hand side and simplify, we obtain the equation

$$0 \;=\; 2\,b^2 \,-\, 2\,bc\cos\,\alpha\ .$$

Solving this for $\cos\,\alpha$, we obtain the desired equation.■

Why include this derivation? At some points in the exercises we shall use this formula, and therefore we need some sort of proof for it. In the formal definition of angle measurement, one takes the cosine function to be defined analytically by the standard infinite series and not by the intuitive formula given above. Thus we should either check or assume that the standard elementary formula for the cosine of an acute vertex angle for a right triangle is given by the usual equation.

One can also retrieve the formula $\sin\,\alpha \;=\; (a/c)$ from such considerations. Specifically, the formula above implies that a/c is equal to the cosine of $\frac{1}{2}\pi - \alpha$, so we only need to give an analytic proof that $\sin\theta \;=\; \cos\left(\frac{1}{2}\pi - \theta\right)$. One quick way to do this is to look at the sum formula for the the cosine of $\frac{1}{2}\pi - \theta$.■