ISOVARIANT HOMOTOPY THEORY AND DIFFERENTIABLE GROUP ACTIONS

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Introduction

This is an expanded version of notes for lectures I had planned to give at the Korea Advanced Institute of Science and Technology during the Seventh KAIST Mathematics Workshop in Algebra and Topology at the Korea Advanced Institute of Science and Technology in Taejon, Korea, from August 11 to August 14, 1992.

There are three parts, with each corresponding to one of the planned lectures. The first two discuss joint work with G. Dula on the foundations of isovariant homotopy theory, the applications of this work to classification problems for smooth manifolds with (smooth) group actions and its relation to work on equivariant surgery over the past two decades; some of the results in the second part have been obtained independently by M. Dawson. The third part discusses joint work with S. Kwasik on a somewhat different but related topic; namely, differentiable actions of finite groups on homology 3-spheres. One common theme relating the second and third parts is the problem of adapting equivariant surgery to cases where a standard technical condition (the *Gap Hypothesis*) does not hold. A second relationship is that the 3-dimensional questions exhibit some basic features of higher-dimensional problems with certain technical simplifications. Finally, advances in the geometrization theory of 3-manifolds over the past two decades have suggested that suitably pseudo-geometric manifolds in higher dimensions form an especially promising subject for future research in geometric topology and transformation groups; perhaps ideas resembling those of Part III will lead to progress in this direction.

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