What does/did the Integral Sign represent?

Some comments on the origin of the usual integral sign notation

$$\int_a^b f(x)\,dx$$

might be useful in understanding the ways in which integrals arise in questions of independent interest. We begin with the limit formula as stated on text page 125 of Guichard (this is page 137 in the `pdf` file):

$$\int_a^b f(x)\,dx \quad = \quad \lim_{n\to\infty} \sum_{i=0}^{n-1} f(t_i)\,dt\ , \quad \text{where} \quad \Delta t \quad = \quad \frac{b-a}{n}$$

If we view the Riemann sums on the right as approximations to the area under the curve $y = f(x)$ for $a \leq x \leq b$, then the sum is actually the sum of the areas of n rectangles of width Δt, and the crucial fact is that these converge to a limiting value (the "actual area") as $n \to \infty$. The integral symbol is a version of the essentially obsolete letter $\int$ which is now written as s, and it was first employed to convey the idea that the integral is a continuous sum of the quantities $f(x)\,dx$, which can be viewed as the areas of rectangles whose vertical sides have length $f(x)$ and whose horizontal sides have a so-called *infinitesimally thin width* which is denoted by dx.

Note on infinitesimals. The idea of working with infinitesimally small quantities turns out to be very useful in working with problems in the sciences and engineering, both as a tool for finding solutions and also for explaining them. However, ever since the invention of calculus mathematicians and others have recognized that justifying the use of such objects in a simple, logically rigorous fashion is at best challenging and at worst not feasible. These difficulties are one reason that mathematicians ended up formulating concepts like limits with δ and ε terminology. About 50 years ago some mathematicians (most notably Abraham Robinson) finally managed to construct a logically sound theory of infinitesimals, and the calculus text by J. Keisler (cited in Guichard) actually develops calculus in this fashion. However, for a variety of reasons it seems unlikely that infinitesimals will replace deltas and epsilons in the mainstream approach to calculus any time soon.