Antiderivatives

Given y', find y.

Since $\frac{d}{dt}(y+C) = \frac{dy}{dt}$, only unique up to adding a constant. If we know $y(t_0)$, we can often find C.

EXAMPLE Find y so that

$y' = x$, $y(1) = 1$.

SOLUTION Find some z so $\frac{dz}{dx} = x$.

$\frac{d}{dx} x^2 = 2x$, so $\frac{d}{dx}\left(\frac{x^2}{2}\right) = x$. Hence

$z = \frac{x^2}{2} + C$ is the general solution.

Now y has this form at $y(1)=1$.

So $1 = y(1) = \frac{1^2}{2} + C$

Solve for C : $1 = \frac{1}{2} + C \Longrightarrow C = \frac{1}{2}$.

Hence $y = \frac{1}{2}(x^2+1)$.

Suppose now $y' = x^2 + x$.

$\frac{d}{dx}\left(\frac{x^3}{3}\right) = x^2$, $\frac{d}{dx}\left(\frac{x^2}{2}\right) = x$, so

$$y = \frac{x^3}{3} + \frac{x^2}{2} + C$$

Find specific function when $y(1) = 1$.

$1 = y(1) = \frac{1}{3} + \frac{1}{2} + C = \frac{5}{6} + C \Longrightarrow$

$\frac{1}{6} = C \Longrightarrow y = \frac{x^3}{3} + \frac{x^2}{2} + \frac{1}{6}$.