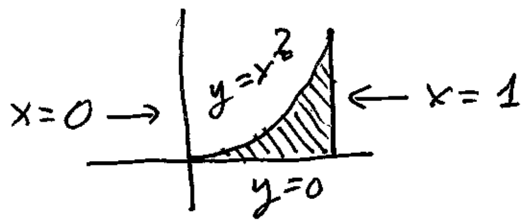
Typical area problems

1. Find the area under the parabola $y = x^2$ where $0 \le x \le 1$.



SKETCH

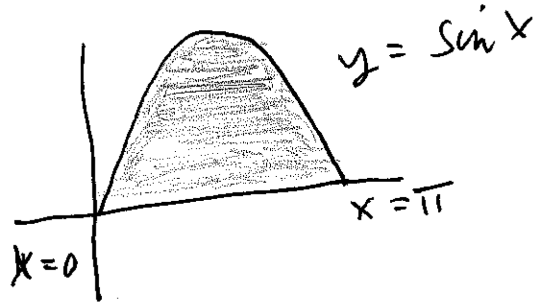
$$A = \int_0^1 x^2\,dx. \quad \frac{d}{dx}\frac{x^3}{3} = x^2, \text{ so}$$

$$\int_0^1 x^2\,dx = \left.\frac{x^3}{3}\right|_0^1$$

← since we take differences in values at 1 and 0, it doesn't matter which anti-derivative we choose.

$$= \frac{1}{3} - 0 = \boxed{\frac{1}{3}}$$

2. Find the area under the sine curve $y = \sin x$ where $0 \le x \le \pi$.



$A = \int_0^{\pi} \sin x \, dx$. Now $\frac{d}{dx}(-\cos x) = \sin x$,

So $-\cos x$ is an antiderivative.

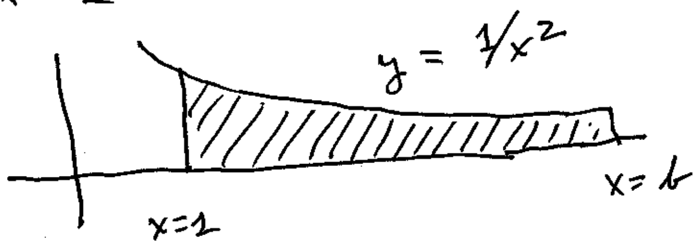
$$\int_0^{\pi} \sin x \, dx = -\cos x \Big|_0^{\pi} = -\cos\pi + \cos 0 =$$

$$-(-1) + 1 = \boxed{2}.$$

So the are in this case is 2.

3. Find the area under $y = \frac{1}{x^2}$ from

$x = 1$ to $x = b$.



$$\frac{d}{dx} x^k = k x^{k-1} \text{ if } k \text{ is an integer (at least)}$$

Say $k-1=-2$. Then

$k=-1$ and we get $\frac{d}{dx}\left(-\frac{1}{x}\right)=\frac{1}{x^2}$.

Hence $A = \int_1^b \frac{dx}{x^2} = -\frac{1}{x}\Big|_1^b =$

$$-\frac{1}{b}+\frac{1}{1}=\left(1-\frac{1}{b}\right).$$

Notice that the area goes to 1 as $b \to \infty$.