Velocity and acceleration

Object moving in a line.

$x(t)$ = position at time t

$v(t)$ = velocity $= \frac{dx}{dt}$

$a(t)$ = acceleration $= \frac{dv}{dt} = \frac{d^2x}{dt^2}$.

Simplest nontrivial example

Falling object — constant gravitational acceleration $-g$ (downward)

Then $v = \int a(t)\,dt = \int -g\,dt = v_0 - gt$

integration constant, initial velocity

$x = \int v(t)\,dt = \int (v_0 - gt)\,dt =$

$v_0 t - gt + x_0$

initial position, another constant

Total distance traveled

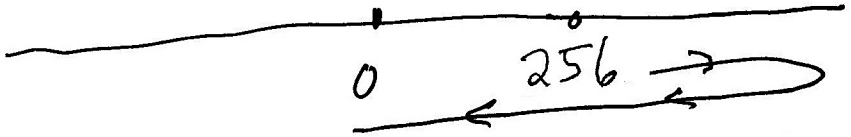
Integrate $|v(t)|$ rather than $v(t)$.

Example $g = 32$ (gravitational acceleration, US units)

$v_0 = 240$. Then $v(t) = 240 - 32t$.

Say $x(0) = 256$.

Find the time $T > 0$ when $x(T) = 0$ and the total distance traveled.



$$x(t) = 256 + 240t - 16t^2.$$

Solve for T:

$$x(t) = x(0) + v(0)t - 16t^2$$
$$= 256 + 240t - 16t^2$$
$$= 16(16 + 15t - t^2) =$$
$$= 16(-1)(t^2 - 15t - 16) =$$
$$= -16(t+1)(t-16).$$

So $x(t) = 0$ if $t = -1, 16$

↑ DISCARD SINCE $T > 0$

$$T = 16$$

Total distance

$$\int_0^{16} |v(t)|\,dt = \int_0^{16} |240 - 32t|\,dt$$

Split into two pieces where $240 - 32t > 0$
$240 - 32t < 0$.

Need to solve $240 - 32t_1 = 0$

$$\boxed{t_1 = 7.5}$$

Hence the distance is $\int_0^T |x(t)|\,dt$ with $x(T)=0$.

$$\int_0^{7.5} (240-32t)\,dt + \int_{7.5}^{16} (32t-240)\,dt$$

since $240-32t<0$ here.

$$= 240t - 16t^2 \Big|_0^{7.5} + 16t^2 - 240t \Big|_{7.5}^{16} =$$

$$(900-0) + \big((4096-3840)-(-900)\big) =$$

$$1800 + 256 = 2056.$$

Basic skill is integrating $\int_a^b |f(x)|\,dx$ —

often useful to split into pieces where $f>0$, $f<0$.
How to split? Look at points where $f=0$.