# Average Values of Continuous Functions

## Finite average

Given $a_1, \dots, a_n$ the average value

$$a_{AVERAGE} = \frac{1}{n}\left(\sum_{i=1}^{n} a_i\right).$$

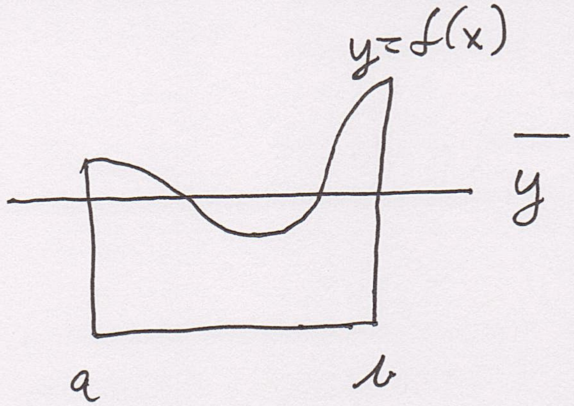
Given function $f(x)$ on $[a, b]$

the average value

$$\overline{y} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

or

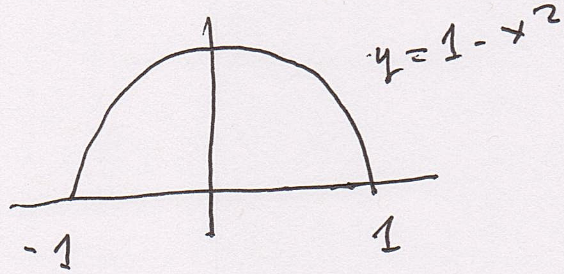
$$\overline{y}\cdot(b-a) = \int_a^b f(x)\,dx$$




$\overline{y} \cdot (b-a) = \int_a^b f(x)\,dx.$

Area under curve $y = f(x)$ between a & b = area of rectangle with same base, height $= \overline{y}$.

<u>Example</u> Find the average value of $y = 1 - x^2$ for $-1 \leq x \leq 1$.




$$\overline{y} = \frac{1}{1-(-1)} \int_{-1}^{1} (1-x^2)\,dx =$$

$$\frac{1}{2}\left(x - \frac{x^3}{3}\right)\Bigg|_{-1}^{1} = \frac{1}{2} \cdot 2\left(1 - \frac{1}{3}\right) = \frac{2}{3}$$

# Uniformly accelerated motion

$v(t) = 128 - 32t$

Find the average speed from $t=0$ to $t = 10$.

Let $w(t) = |v(t)|$, speed.

$$\overline{w} = \frac{1}{10} \int_0^{10} |v(t)|\,dt.$$

Break integral into two pieces where $v \geq 0$, $v \leq 0$.

Breaking point is where $v = 0$

SOLVE $0 = 128 - 32t \Rightarrow t = 4$.

So we have

$$\text{Total distance} = \int_0^{10} |128-32t|\,dt$$

$$= \int_0^4 (128-32t)\,dt + \int_4^{10} (32t-128)\,dt =$$

here $v(t) > 0$ here $v(t) < 0$

$$128t - 16t^2 \Big|_0^4 + 16t^2 - 128t \Big|_4^{10} =$$

$$(512-256) + (1600-256) - (1280-512)$$

$$= 256 + 1344 - 768 = 832.$$

So the average speed is

$$\frac{1}{10} \cdot 832 = 83.2.$$

Contrast to average velocity

$$\overline{v} = \frac{1}{10}\int_0^{10}(128-32t)\,dt =$$

$$\frac{1}{10}\left(128t - 16t^2\right)\Big|_0^{10} =$$

$$\frac{1}{10}\left(1280 - 1600\right) =$$

$$\frac{1}{10} \cdot 320 = -32.$$

So average velocity is in opposite direction ~~to~~ initial velocity.