Hyperbolic Functions

NAME: hyperbolic cosine hyperbolic sine
SYMBOLS: $\cosh x$, $\sinh x$ such that
PRONOUNCED: "COSH" "SINCH"

$$\cosh^2 x - \sinh^2 x = 1 \qquad \cosh 0 = 1 \quad \sinh 0 = 0$$

$$\frac{d}{dx}\cosh x = \sinh x \qquad \frac{d}{dx}\sinh x = \cosh x$$

Explicit Formulas

$$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right),$$

$$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right).$$

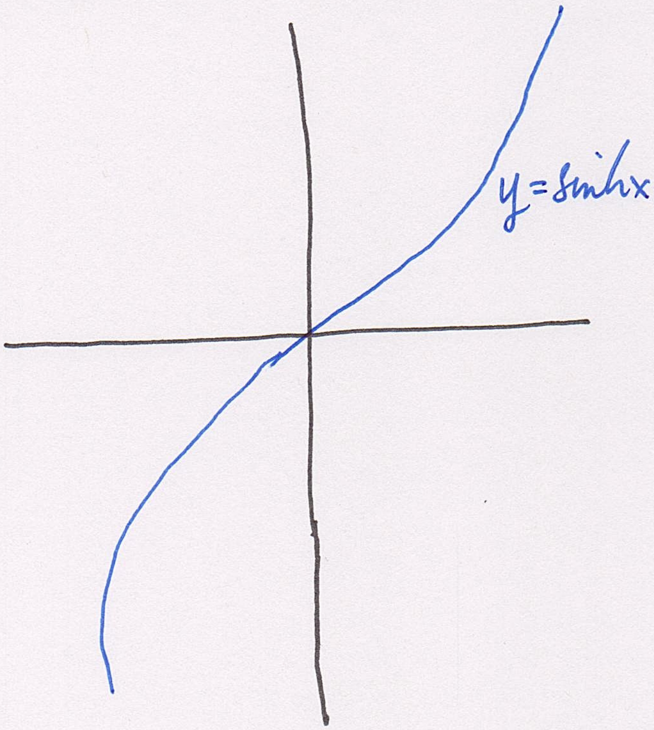
Differentiation formulas are easy to check, as are $\cosh 0 = 1$, $\sinh 0 = 0$.

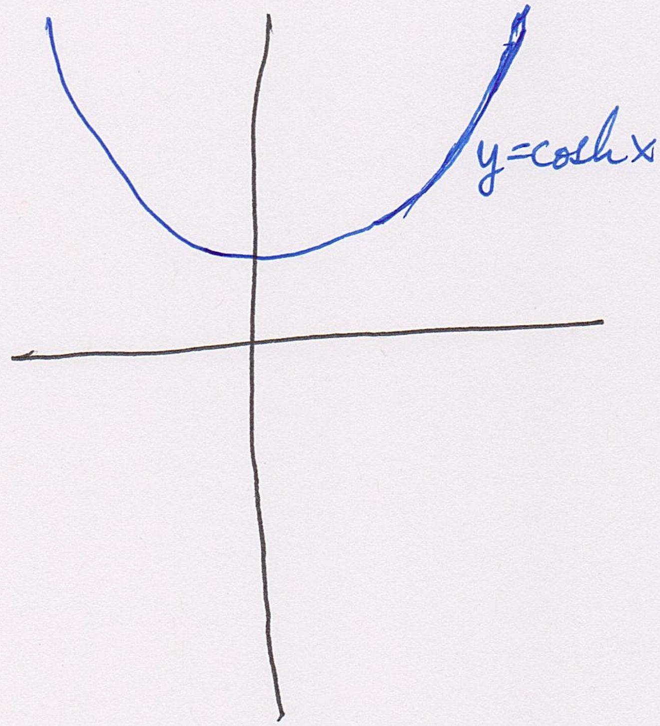
$$\cosh^2 x - \sinh^2 x = \frac{1}{4}\left(e^{x} + e^{-x}\right)^2 - \frac{1}{4}\left(e^{x} - e^{-x}\right)^2$$

$$= \frac{1}{4}\left(\cancel{e^{2x}} + 2 + \cancel{e^{-2x}}\right) - \frac{1}{4}\left(\cancel{e^{2x}} - 2 + \cancel{e^{-2x}}\right) =$$

terms cancel

$$\frac{1}{2} - \left(-\frac{1}{2}\right) = 1.$$





$$\lim_{x\to\pm\infty} \sinh x = \tfrac{1}{2}(e^{x}-e^{-x}) = \pm\infty$$

$$\sinh(-x) = -\sinh x$$

$$\cosh x \geq 1$$

$$\lim_{x\to\pm\infty} \cosh x = +\infty$$

$$\cosh(-x) = \cosh x.$$

graph = catenary = eqn. of hanging chain.

OTHER HYPERBOLIC FUNCTIONS.

$$\tanh x = \frac{\sinh x}{\cosh x} \ (\text{all } x) \qquad \coth x = \frac{\cosh x}{\sinh x} \ (x \neq 0).$$

$$\operatorname{sech} x = \frac{1}{\cosh x} \ (-''-) \qquad \operatorname{csch} x = \frac{1}{\sinh x}. \ (-''-)$$

More identities

① $$\frac{d}{dx}\tanh x = \frac{d}{dx}\frac{\sinh x}{\cosh x} = \frac{\cosh x \cosh x - \sinh x \sinh x}{\cosh^2 x}$$

$$= \frac{1}{\cosh^2 x} = \operatorname{sech}^2 x.$$

② $$1 - \tanh^2 x = \operatorname{sech}^2 x.$$

③ $$\frac{d}{dy}\,\mathrm{inv}\sinh y = \frac{1}{\cosh(\mathrm{inv}\sinh y)} =$$

So we want $\cosh u$ where $u = \mathrm{inv}\sinh y$
or $y = \sinh u$.

But this means that $\cosh u = \sqrt{1+y^2}$.

Hence $$\frac{d}{dy}\,\mathrm{inv}\sinh y = \frac{1}{\sqrt{1+y^2}}.$$

④ Find $\cosh\frac{x}{2}$.

Use

Exercise 6, p. 186:

$$\cosh 2u = \cosh^2 u + \sinh^2 u =$$
$$\cosh^2 u + (\cosh^2 u - 1) =$$
$$2\cosh^2 u - 1. \quad \text{Let } u = \frac{x}{2}$$

$$\cosh x = 2\cosh^2 \frac{x}{2} - 1$$

$$\cosh^2 \frac{x}{2} = \frac{\cosh x + 1}{2}$$

$$\cosh \frac{x}{2} = \sqrt{\frac{\cosh x + 1}{2}}.$$

(5) What is the range of $\tanh x$?

Let $y = \sinh x$. Then $\tanh x = \dfrac{y}{\sqrt{1+y^2}}$

This goes to ± 1 as $y \longrightarrow \pm\infty$.