Finding antiderivatives

Historically, one main topic of integral calculus courses has been an assortment of techniques for expressing many indefinite integrals $\int f(x)\,dx$ in terms of the standard functions in calculus (polynomials, $+, -, \times, \div, \sqrt[n]{\ }$, exponential, logarithmic, trigonometric and inverse trig functions). Less emphasis here for a few reasons:

1. For really challenging problems, most users are likely to use tables or software (Maple, Mathematica, etc.)

(2) There are many important choices of $f(x)$ for which one cannot write $\int f(x)\,dx$ in terms of standard functions. For example $\sin x^2$, e^{ax^2}, "most" expressions involving $\sqrt[n]{\ }$.

[See http://math.ucr.edu/~res/math9C/nonelementary_integrals.pdf for a discussion, at the 9C level in the final remark.]

However, some coverage is needed because

(1) there are important uses of such techniques and their applications to special cases,

(2) even with tables, sometimes a bit of knowledge about such techniques can be helpful.

Our emphasis will be on uses of techniques to solve problems of independent interest.

Examples 1. How integral calculus relates to finding areas of (regions bounded by) circles, circular sectors, etc.

2. Equations which model population growth.

3. Some questions about AC electrical circuits, other "vibrational" phenomena.
oscillation

Powers of sine and cosine functions

Key fact $\sin^k x + \cos^k x$ can be written as sums of the form

$$\frac{a_0}{2} + a_1 \cos x + b_1 \sin x + \ldots + a_k \cos kx + b_k \sin kx.$$

Follows from trig identities (won't elaborate but will give some examples)

Find $\int \sin^2 x\, dx$

Use trig identity: $\cos 2x = \cos^2 x - \sin^2 x =$

$(1 - \sin^2 x) - \sin^2 x = 1 - 2\sin^2 x$. Hence

$$\boxed{\sin^2 x = \frac{1 - \cos 2x}{2}.}$$ So

$\int \sin^2 x\, dx = \frac{1}{2}\int (1 - \cos 2x)\, dx =$

$\frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x\, dx =$

let $u = 2x$, so $du = 2\,dx$

$\frac{1}{2}\int dx - \frac{1}{2}\int \cos u \cdot \frac{1}{2} du =$

$\frac{1}{2}\int dx - \frac{1}{4}\int \cos u\, du =$

$\frac{x}{2} - \frac{1}{4}\sin u + C \underset{u=2x}{=}$

$\frac{x}{2} - \frac{1}{4}\sin 2x + C.$

Typical use in AC circuit problem

Power = $(\text{current})^2 \cdot (\text{resistance})$

$(A \sin 2\pi k t)^2 \cdot R$ constant

k = number of cycles per second

U.S. $k=60$
Europe $k=50$

A = amplitude, max current

Power = $\frac{d}{dt}$(Energy), so

$$\text{Energy} = \int_{t_0}^{t_1} (\text{Power}) \cdot dt$$

Find total energy use in 1 second

$t_0 = 0$, $t_1 = 1$; this $\Leftrightarrow$ average power

(for example, gives heat from an electric heating device, light for old style light bulb).

COMPUTE $\int_0^1 A^2 R \sin^2 2k\pi t \; dt$

Pull the constant out, getting

$$A^2R\int_0^1 \sin^2 2k\pi t\, dt.$$

Find anti-derivative $\sin^2 2k\pi t$

Now let $u = 2k\pi t$, so

$du = 2k\pi\, dt$.

Then $\int \sin^2 2k\pi t\, dt = \frac{1}{2k\pi}\int \sin^2 u\, du =$

$$\frac{1}{2k\pi}\left[\frac{u}{2} - \frac{\sin 2u}{4}\right] + C$$

$$= \frac{1}{\cancel{2k\pi}}\cdot\frac{\cancel{2k\pi} t}{2} - \frac{\sin 4k\pi t}{8k\pi} + C$$

Hence the definite integral equals

$$A^2R\left(\frac{t}{2} - \frac{\sin 4k\pi t}{8k\pi}\Bigg|_0^1\right) = \frac{A^2R}{2}$$

values at 1 + 0 are equal!

Same average power as DC current of $\frac{A}{\sqrt{2}}$

[Root Mean Square]