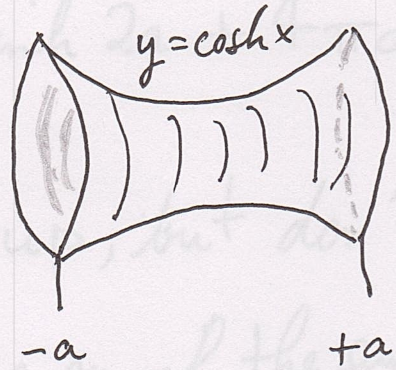
Surface area examples

Catenoid Rotate $y = \cosh x$

$-a \le x \le a$

about the x-axis.



Formula: $A = \int_{-a}^{a} 2\pi y \sqrt{1+(y')^2}\, dx =$

$$2\pi \int_{-a}^{a} \cosh x \sqrt{1+\sinh^2 x}\, dx = 2\pi \int_{-a}^{a} \cosh^2 x\, dx =$$

$$\cosh^2 x = \frac{1}{2}(\cosh 2x + 1)$$

$$\pi \int_{-a}^{a} (\cosh 2x + 1)\, dx = \pi \int_{-a}^{a} \cosh 2x\, dx + 2\pi a$$

$$\boxed{\int \cosh 2x\, dx = \int \cosh u \cdot \frac{du}{2} = \frac{1}{2}\int \cosh u\, du}$$

Let $u = 2x$. Then the integral is

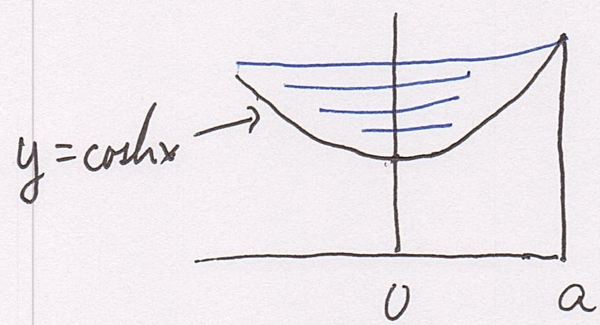
$$\frac{\pi}{2}\int_{-2a}^{2a} \cosh u \, du + 2\pi a =$$

$\left[\begin{array}{l}\text{If } x = \pm a \\ \text{then} \\ u = 2x = \pm 2a.\end{array}\right]$

$$\frac{\pi}{2} \sinh u \Big|_{-2a}^{2a} + 2\pi a =$$

$$\pi \sinh 2a + 2\pi a.$$

Set up, but don't evaluate, the integral for the area of the surface formed by rotating $y = \cosh x$ about the y-axis. $0 \le x \le a$



$$A = \int_0^a 2\pi x \sqrt{1+(y')^2}\, dx =$$

$$2\pi \int_0^a x\sqrt{1+\sinh^2 x}\, dx = 2\pi \int_0^a x \cosh x\, dx$$

How to evaluate $\int x \cosh x\, dx =$

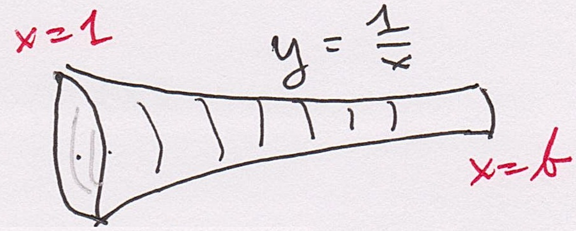
$u = x$
$v = \sinh x$

$x \sinh x - \int \sinh x\, dx = x \sinh x - \cosh x.$

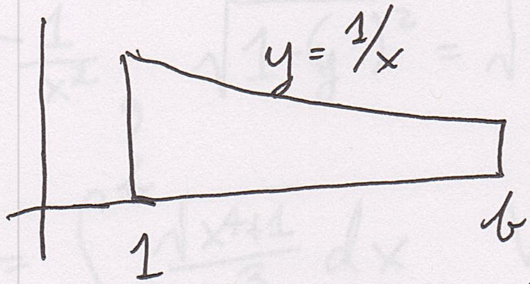
Fill in the details!!!

ESTIMATING SURFACE AREAS

Take the surface formed by rotating $y = \frac{1}{x}$, from $x=1$ to $x=b$, about the x-axis. Show that the limit as $b \to \infty$ is equal to $+\infty$.



In contrast, the volumes of the solids of revolution generated by



go to a finite limit as $b \to \infty$ (see the notes for Ch. 8).

So the solid of revolution corresponding to $1 \leq x \leq \infty$ has finite volume, but the surface bounding this solid has infinite total surface area!

$$\text{Area} = \int_1^b 2\pi y \sqrt{1+(y')^2}\, dx.$$

$$y' = \frac{-1}{x^2}, \quad \sqrt{1+(y')^2} = \sqrt{\frac{x^4+1}{x^4}} = \frac{\sqrt{x^4+1}}{x^2}.$$

So $A_b = \int_1^b \frac{\sqrt{x^4+1}}{x^3}\, dx$ Want to show this $\longrightarrow \infty$ as $b \to \infty$. Evaluating the integral is hard, but it's easier to estimate it:

$$\sqrt{x^4+1} \geq x^2$$

$$A_b = \int_1^b \frac{\sqrt{x^4+1}}{x^3}\, dx \geq \int_1^b \frac{x^2}{x^3}\, dx = \int_1^b \frac{dx}{x}$$

$$= \log b.$$

Since $\lim_{b\to\infty} \log b = \infty$, we also have

$$\lim_{b\to\infty} A_b = \infty.$$

Read and understand the last example in Section 11.5 of the text!

Also understand the derivation of the surface area foormula for a sphere.