

Problem Find the surface area
 for the surface of revolution obtained
 by rotating $y = \cosh x$ about the y -axis
 ($0 \leq x \leq b$).

$$\left[\begin{array}{l} \text{Recall } \cosh^2 - \sinh^2 = 1 \\ \Rightarrow 1 + \sinh^2 = \cosh^2 \end{array} \right]$$

Start with the general formula:

$$A = 2\pi \int_0^b x \sqrt{1+(y')^2} dx \quad \left[\begin{array}{l} y = \cosh x \\ y' = \sinh x \\ 1+(y')^2 = \cosh^2 x \end{array} \right]$$

So $A = 2\pi \int_0^b x \cosh x dx$. Integrate by parts

$$\int \underbrace{x}_{u} \underbrace{\cosh x}_{dv} dx = x \sinh x - \int \sinh x dx = x \sinh x - \cosh x.$$

$$\text{Hence } A = 2\pi (x \sinh x - \cosh x) \Big|_0^b = 2\pi (b \sinh b + 1 - \cosh b).$$