Answers to selected exercises from Colley, Section 1.2

8. The answer is $(-6, 0, 21)$.

14. The parametric equations are $x = 12 + 5t$, $y = -2 - 12t$, and $z = t$.

16. The parametric equations are $x = 2 + t$, $y = 1 - 2t$, and $z = 2 + 3t$.

22. Set each of the three ratios equal to t and solve for the coordinates. The parametric equations found in this way are $x = 2 + 5t$, $y = 3 - 2t$, $z = -1 + 4t$.

23. One way to do this is to show that two points on the first line also lie on the second. Since there is only one line joining two points, this means that the two lines must be the same. We obtain the two points on the first line by setting the common ratio equal to 0 and 1. These choices yield the points $(2, 1, 0)$ and $(5, 8, 5)$.

To prove that the first point lies on the line, we need to show that the three ratios

$$\frac{2+1}{-6}, \qquad \frac{1+6}{-14}, \qquad \frac{0+5}{-10}$$

are all equal. Direct calculation shows that they are all equal to $-\frac{1}{2}$, so this is indeed the case. Next, we have to do the analogous check for the ratios

$$\frac{5+1}{-6}, \qquad \frac{8+6}{-14}, \qquad \frac{5+5}{-10}$$

and in this case we see that all three ratios are equal to -1. Thus both points from the first line also lie on the second, so that the lines must be the same.

34. Let $x_1(t), y_1(t), z_1(t)$ denote the parametric equations for the first line and let $x_2(t), y_2(t), z_2(t)$ denote the parametric equations for the second. As noted in the text, the two lines meet if and only if there are numbers u and v such that $x_1(u) = x_2(v)$, $y_1(u) = y_2(v)$, and $z_1(u) = z_2(v)$. If we solve these equations for u and v we get $u = -1$ and $v = 2$, so that there is a common point and it is $(1, 0, -1)$.