Answers to selected exercises from Colley, Section 2.7

2. FALSE. The formulas defining the coordinates are only meaningful if $y \neq 0$ and $x \neq -y$; on the other hand, the formula can be used to define the function if $x = y$ and both are nonzero.

8. TRUE. If $f(x, y)$ is the original function, then the graph is a level set (in fact, the zero set) for the function $G(x, y, z) = z - f(x, y)$.

9. FALSE. The sphere defined by the equation $x^2 + y^2 + z^2$ is a counterexample - if $x^2 + y^2 < 1$, then both $(x, y, \pm\sqrt{1 - x^2 - y^2})$ lie in the set, while a graph has the property that for a given (x, y) in the domeain there is only one z such that (x, y, z) lies in the graph.

10. FALSE. The limit does not exist, for if we change variables to polar coordinates we see that the expression is given by $\cos^2\theta - 2\sin^2\theta = 3\cos^2\theta - 2$. For every $r > 0$ this takes every value between 1 and -2, so in particular there is no $h > 0$ such that the value z of the function is between $\pm\frac{1}{2}$ when the distance from (x, y) to $(0, 0)$ is less than h.