Answers to selected exercises from Colley, Section 3.6

3. The speed $S(t)$ is given by $|\mathbf{x}'(t)|$ and its square is given by $\mathbf{x}' \cdot \mathbf{x}'$. Now $S(t)$ is constant if and only if $S(t)^2$ is constant, which is the same as saying that

$$\frac{d}{dt}\left(\mathbf{x}' \cdot \mathbf{x}'\right) \;=\; 0\ .$$

By the Leibniz rule for differentiating dot products, the latter is equal to $2\,\mathbf{x}'(t) \cdot \mathbf{x}''(t)$, and this is zero if and only if the acceleration vector is perpendicular to the (nonzero) velocity vector. Hence the speed is constant if and only if the latter condition holds.