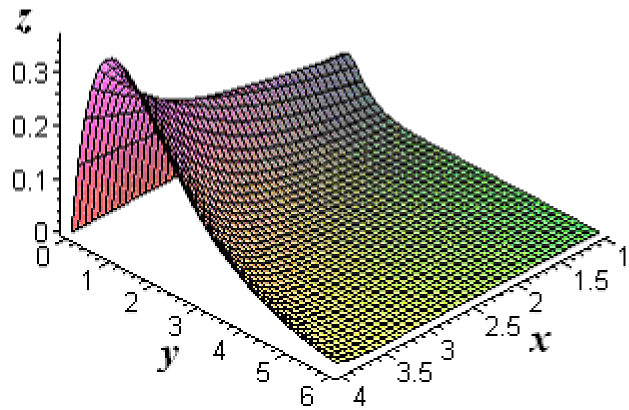
Physical interpretation of the gradient

In the lectures we have discussed some physical interpretations of the gradient. One is given in terms of the graph of some function $z = f(x, y)$, where f is a reasonable function – say with continuous first partial derivatives. In this case we can think of the graph as a surface whose points have variable heights over the xy – plane. An illustration is given below.



$$z(x, y) = y\exp(-y(5-x))$$

(***Source:*** http://www.math.uri.edu/Center/workht/calc3/graphs1.html)

If, say, we place a marble at some point (x, y) on this graph with zero initial force, its motion will trace out a path on the surface, and in fact it will choose the direction of steepest descent. This direction of steepest descent is given by the ***negative*** of the ***gradient*** of f. One takes the negative direction because the height is decreasing rather than increasing.

Using the language of vector fields, we may restate this as follows: For the given function $f(x, y)$, gravitational force defines a vector field $\mathbf{F}$ over the corresponding surface $z = f(x, y)$, and the initial velocity of an object at a point (x, y) is given mathematically by $-\nabla f(x, y)$.

The gradient also describes directions of maximum change in other contexts. For example, if we think of f as describing the temperature at a point (x, y), then the gradient gives the direction in which the temperature is increasing most rapidly.

Finally, here are some additional online references for gradients:

http://www.slideshare.net/leingang/lesson-15-gradients-and-level-curves/

http://www.youtube.com/watch?v=CmoxhxK8Org

http://www.youtube.com/watch?v=JJcItzIJWWQ&feature=related