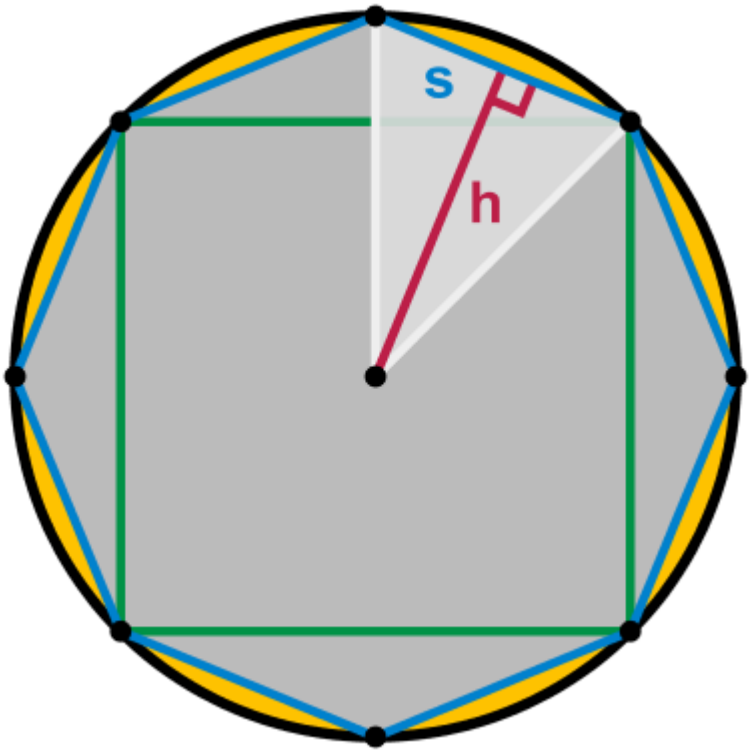
An elementary example involving the limit concept

It is likely that one of the earliest limit problems was to determine the area of the solid region bounded by a circle. One way to attack this problem is to approximate the given region by a solid region bounded by regular polygons whose vertices lie on the circle. As one increases the number of sides of the regular polygon, the area of the solid region bounded by the polygon increases, and intuitively speaking it seems apparent that the limit of these values should be the area of the region bounded by the circle. Here is a simple illustration involving a square and a regular octagon.



(Source: http://commons.wikimedia.org/wiki/Image:Archimedes_circle_area_proof_-_inscribed_polygons.png)

One reason for suspecting the statement about areas is that every point in the circular region lies in the region bounded by all regular n – gons as above, provided n is sufficiently large. We can rephrase the latter as saying that n is sufficiently close to infinity. Now the areas bounded by the regular polygons increase as the number of vertices increase, so it follows that we should have that the areas of the polygonal regions get close and stay close to the area of the circular region provided n is sufficiently large (or close to infinity, or equivalently as n approaches infinity). At the site

http://www.karlscalculus.org/calc12_2.html

there is an animated drawing (Figure $\mathbf{12.2-1}$) showing the limiting behavior, and there is also a table of values (Table $\mathbf{12.2-1}$) which show how the perimeters of regular n –

gons approach the circumference of the circle as n approaches infinity. Another example showing how the areas of the polygonal regions approach the area bounded by the circle appears in the following online site:

http://www.math.psu.edu/courses/maserick/circle/circleapplet.html

Here are two parts of a YouTube series which reviews the basic facts about limits in general:

http://www.youtube.com/watch?v=l8n0RD6RQFQ&feature=related

http://www.youtube.com/watch?v=QCHJOctrhd0&feature=related