NAME: ______________________________

Mathematics 10A–001, Fall 2008, Examination 1

Answer Key

1. [*20 points*] Find the cross product $\mathbf{a} \times \mathbf{b}$ where $\mathbf{a} = (1, 1, 0)$ and $\mathbf{b} = (0, 0, 1)$.

SOLUTION

The cross product is given by the 3×3 determinant formula:

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

In coordinates this can be rewritten as

$$\left(\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} \right)$$

and if we evaluate the 2×2 determinants giving the coordinates we obtain the vector $(1, -1, 0)$.

2. [*10 points*] Find the value of x such that the vectors $(x, 3, 2, 1)$ and $(1, 2, 3, 4)$ are perpendicular.

SOLUTION

The vectors are perpendicular if and only if their dot product is zero. But this dot product is

$$x \ + \ (3 \cdot 2) \ + \ (2 \cdot 3) \ + \ (1 \cdot 4) \quad = \quad x + 16$$

so the dot product is zero if and only if $x = -16$.

3. [*20 points*] Let P be the plane defined by the equation $x + y + z = 1$, let $\mathbf{x} = (2, 3, 4)$, and let L be the line through $\mathbf{x}$ which is perpendicular to P. Find a second point on L.

SOLUTION

The perpendicular direction to the plane is given by the coefficients of the variables in the defining equation, so it is a nonzero multiple of $(1, 1, 1)$, and therefore the remaining points on L all have the form $(2 + t, 3 + t, 4 + t)$ for some $t \neq 0$. Any particular choice of t will give a correct answer.

4. [*15 points*] The point X in the coordinate plane has polar coordinates $(\sqrt{3}, 5\pi/6)$. Find its rectangular coordinates. State your answer in a form not involving trigonometric functions. [*Hints:* $5\pi/6$ radians $= 150° = 90° + 60°$, $\cos 60° = \frac{1}{2}$ and $\sin 60° = \frac{1}{2}\sqrt{3}$.]

SOLUTION

We have $x = r\cos\theta$ and $y = r\sin\theta$, where $r = \sqrt{3}$ and $\theta = 150°$. The trigonometric functions of obtuse angles are given by $\cos(90° + \theta) = -\sin\theta$ and $\cos(90° + \theta) = \cos\theta$, so that $\cos 150° = -\sin 60°$ and $\sin 150° = \cos 60°$. If we substitute into the polar coordinate formulas we find that $x = -3/2$ and $y = \frac{1}{2}\sqrt{3}$.

Note. The sum formulas for sine and cosine are **NOT** given by $\sin(\alpha + \beta) = \sin\alpha + \sin\beta$ or $\cos(\alpha + \beta) = \cos\alpha + \cos\beta$, and these incorrect formulas will almost always yield the wrong answer **!!!**

5. [*15 points*] Find the rectangular coordinate equation for the curve given by the polar coordinate equation $r = 2\cos\theta + 4\sin\theta$.

EXTRA CREDIT. [*10 points*] Describe this curve in one line or less. You need not give reasons for your answer.

SOLUTION

Multiply both sides by r to obtain $r^2 = 2r\cos\theta + 4r\sin\theta$. This translates into the rectangular equation $x^2 + y^2 = 2x + 4y$.

EXTRA CREDIT

The equation defines a circle. Its center is $(1, 2)$ and its radius is $\sqrt{5}$.

6. [*20 points*] Given that the vector $\mathbf{v} = (x, y, z)$ satisfies the equation $2\mathbf{v} + (3, 11, 7) = (5, 9, 17)$, find (the coordinates of) $\mathbf{v}$.

SOLUTION

We can rewrite the eqution as $(2x+3, 2y+11, 2z+7) = (5, 9, 17)$, which is equivalent to the system of equations $2x + 3 = 5$, $2x + 11 = 9$, and $2x + 7 = 17$. If we solve this system we find that $\mathbf{v} = (x, y, z) = (1, -1, 5)$.