Math 10A–001
Fall 2008

UPDATED GENERAL INFORMATION — DECEMBER 4, 2008

The following example was worked out in class yesterday:

> *Find the maximum value of the function* $f(x, y, z) = x + y + z$ *on the elliptic paraboloid defined by* $z = 4 - x^2 - y^2$.

The constraint is given by $0 = g(x, y, z) = 4 - x^2 - y^2 - z$, and therefore we need to solve the system of equations given by this and $\nabla(f - \lambda g) = \mathbf{0}$. By definition we have

$$\nabla(f - \lambda g) \quad = \quad (1 + 2\lambda x, 1 + 2\lambda y, 1 + \lambda)$$

and if we set each of the three coordinates equal to zero we can use the equation for the third coordinate to conclude that $\lambda = -1$. Substituting this back into the first two equations we see that

$$0 \quad = \quad 1 - 2x \quad = \quad 1 - 2y$$

so that $x = y = \frac{1}{2}$. Substituting these back into the constraint we see that $z = 4 - \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = 4 - 2 \cdot \frac{1}{4} = 3\frac{1}{2}$. Therefore the maximum occurs at the point $\left(\frac{1}{2}, \frac{1}{2}, \frac{7}{2}\right)$ and the maximum value is $f\left(\frac{1}{2}, \frac{1}{2}, \frac{7}{2}\right) = \frac{1}{2} + \frac{1}{2} + \frac{7}{2} = \frac{9}{2}$.■