ADDITIONAL EXERCISES FOR MATHEMATICS 10B

FALL 2009

Chapter 5

The exercises are organized by sections of the course text (Colley, *Vector Calculus*, Third Edition).

Exercises for Colley, Section 5.1

S1. Evaluate the following iterated integral:

$$\int_0^1 \int_0^2 (x+y)\, dy\, dx$$

S2. Evaluate the following iterated integral, and describe the region R for which the expression represents a double integral over R:

$$\int_0^2 \int_0^1 (1+2x+2y)\, dy\, dx$$

Exercises for Colley, Section 5.2

S1. Use double integrals to find the volume of the solid defined by the inequalities $0 \leq x \leq 4$, $0 \leq y \leq 2$, $0 \leq z \leq 6-2y$.

S2. Is the following equation true or false? Give reasons to support your answer.

$$\int_{-1}^1 \int_{-1}^1 \cos(x^2+y^2)\, dx\, dy \quad = \quad 4 \cdot \int_0^1 \int_0^1 \cos(x^2+y^2)\, dx\, dy$$

S3. Evaluate the following iterated integral:

$$\int_1^2 \int_1^x \frac{1}{(x+y)^2}\, dy\, dx$$

S4. Evaluate the following iterated integral:

$$\int_0^1 \int_0^x \sqrt{1-x^2}\, dy\, dx$$

S5. Evaluate the following iterated integral:

$$\int_0^1 \int_0^{\sqrt{1-y^2}} (x+y)\, dy\, dx$$

S6. Determine the limits of integration for the double integral

$$\int\int_A f(x,y)\, dx\, dy$$

if A is the region bounded by the graphs of $x = -y$ and $x = 2y - y^2$, for both orders of integration. Find the area of A by computing this integral when $f = 1$:

S7. Find the volume of the 3-dimensional region bounded by the surfaces $z = \sin y$, $x = 0$, $x = 1$, $y = 0$, $y = \frac{1}{2}\pi$, and the xy-plane.

Exercises for Colley, Section 5.3

S1. Interchange the order of integration in the following iterated integral:

$$\int_0^1 \int_0^x \sqrt{1-x^2}\, dy\, dx$$

S2. Interchange the order of integration in the following iterated integral:

$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \log_e(1 + x^2y^2)\, dx\, dy$$

S3. Interchange the order of integration in the following iterated integral:

$$\int_0^1 \int_{y^3}^{\sqrt{y}} \tan(xy)\, dx\, dy$$

S4. Evaluate the following iterated integral by interchanging the order of integration:

$$\int_0^1 \int_y^1 \sin x^2\, dx\, dy$$

Exercises for Colley, Section 5.4

S1. Find the volume of the solid bounded by the planes with equations $x = 0$, $y = 0$, $z = 0$ and $2x + 3y + 4z = 12$. [*Hint:* The intersection of this plane with the $xy-$plane is the line joining the points $(6, 0, 0)$ and $(0, 4, 0)$.]

S2. Find the volume of the solid defined by the inequalities $0 \leq x \leq y$, $0 \leq y \leq 1$, $0 \leq z \leq 1 - xy$.

S3. Find the triple integral of the function ze^{x+y} over the cube $0 \leq x, y, z \leq 1$.

S4. Find the iterated integral limits of integration for the region cut by the solid disk $x^2 + y^2 + z^2 \leq 4$ from the cylinder $2x^2 + z^2 = 1$.

S5. Find the iterated integral limits of integration for the region bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y = 4$ and $x - z - y - 1$.

S6. Evaluate the following iterated integral:

$$\int_0^1 \int_0^x \int_0^y (x + yz)\, dz\, dy\, dx$$

S7. Suppose that $f(x, y, z)$ is a reasonable function which is defined for all points in coordinate 3-space, and let W be the region defined by the inequalities $x^2 + y^2 + z^2 \leq 1$ and $\frac{1}{2} \leq z \leq 1$. Express the triple integral of f over W as an interated integral $\int \int \int \ldots dz\, dy\, dx$; in other words, find the limits of integration.

Exercises for Colley, Section 5.5

S1. Let $(x, y) = \mathbf{T}(u, v) = (3u + 2v,\ 3v)$, let R be the set of points in the $xy-$plane that are on or inside the triangle with vertices $(0, 0)$, $(3, 0)$ and $(2, 3)$, and let S be the region in the $uv-$plane which maps to R under $\mathbf{T}$. Describe the region S by (a) drawing a sketch in the $uv-$plane, (b) expressing this in words or algebraic inequalities.

S2. Let $(x, y) = \mathbf{T}(u, v) = (4u + v,\ u + 2v)$, let R be the set of points in the $xy-$plane that are on or inside the parallelogram with vertices $(0, 0)$, $(1, 2)$, $(4, 1)$ and $(5, 3)$, and let S be the region in the $uv-$plane which maps to R under $\mathbf{T}$. Describe the region S by (a) drawing a sketch in the $uv-$plane, (b) expressing this in words or algebraic inequalities.

S3. Evaluate the following integral by changing from rectangular to polar coordinates:

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} y\, dx\, dy$$

S4. Evaluate the following integral by changing from rectangular to polar coordinates:

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} xy\, dy\, dx$$

S5. Evaluate the following integral by changing from rectangular to cylindrical coordinates:

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^4 x\, dz\, dy\, dx$$

S6. Evaluate the following integral by changing from rectangular to spherical coordinates:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx$$

S7. Let $(x, y) = \mathbf{T}(u, v) = \left(\frac{1}{2}(u+v), \frac{1}{2}(u-v)\right)$, and let R be the set of points in the $xy-$plane that are on or inside the square with vertices $(1, 0)$, $(0, 1)$, $(-1, 0)$ and $(0, -1)$. Evaluate the following integral using the change of variables associated to $\mathbf{T}$:

$$\int\int_R 4(x^2+y^2)\, dx\, dy$$

S8. Using the same definitions and procedure as in the previous exercise, evaluate the following integral:

$$\int\int_R 48xy\, dx\, dy$$

S9. Find the Jacobian

$$\frac{\partial(x, y, x)}{\partial(u, v, w)}$$

for the change of variables associated to the following transformation:

$$(x, y, z) \quad = \quad \mathbf{T}(u, v, w) \quad = \quad \left(u(1-v),\ uv(1-w),\ uvw\right)$$

S10. Find the Jacobian

$$\frac{\partial(x, y, x)}{\partial(u, v, w)}$$

for the change of variables associated to the following transformation:

$$(x, y, z) \quad = \quad \mathbf{T}(u, v, w) \quad = \quad \left(4u-v,\ 4v-w,\ u+w\right)$$

S11. Let $\mathbf{T}$ be the change of variables transformation $(x, y) = \mathbf{T}(u, v) = (u^2 - v^2, 2uv)$, let S be the region in the uv-plane defined by $u, v \geq 0$ and $u^2 + v^2 \leq 1$, and let R be the image of S under $\mathbf{T}$. Find the area of R, and also describe the integrals of the functions x and y over D as integrals in u and v, and convert these to polar coordinates using the change of variables $u = r\cos\theta$, $v = r\sin\theta$.

Exercises for Colley, Section 5.6

S1. Using cylindrical coordinates, find the centroid of a right circular cone for which the base has radius r and the height is equal to h.

S2. Using spherical coordinates, find the centroid of a hemispherical solid of radius r with uniform density.

S3. Using spherical coordinates, find the centroid of the solid of uniform density lying between two concentric hemispheres with radii r and R, where $r < R$.

S4. Find the average value of $f(x, y) = e^{x+y}$ on the solid triangular region with vertices $(0, 0)$, $(1, 0)$ and $(0, 1)$. This region is defined by the inequalities $x, y \geq 0$ and $x + y \leq 1$.

S5. Find the average value of $f(x, y, z) = e^{-z}$ on the disk $x^2 + y^2 + z^2 \leq 1$.