UPDATED GENERAL INFORMATION — FEBRUARY 18, 2017

The second in-class examination

The second in-class examination has been postponed to **Friday, February 24.** It will cover the material from Section 6C through Section 7C. Three basic aspects of the subject are particularly important; namely, working examples, correctly formulating the basic definitions and key results, and doing simple proofs and derivations (if an argumment takes more than about a third of a page, it is safe to assume it will not appear on the examination).

Aside from the previously recommended exercises, here are a few further problems worth considering:

1. Let V be a finite dimensional inner product space, and let $E : V \to V$ denote orthogonal (perpendicular) projection onto a subspace W.

(a) Show that $E^2 = E$ and E is self-adjoint.

(b) Suppose that E_1 and E_2 are orthogonal projctions onto W_1 and W_2 satisfying $E_1E_2 = E_2E_1$. Prove that the latter defines perpendicular projection onto $W_1 \cap W_2$.

2. Let V be a finite dimensional inner product space, and let $T : V \to V$ be a linear transformation. Prove that T is invertible if and only if its adjoint T^* is invertible.

3. Let V be a finite dimensional real inner product space. A linear transformation $T : V \to V$is said to be *conformal* if it preserves the (cosines of) angles between nonzero vectors; *i.e.*, if $\mathbf{x}$ and $\mathbf{y}$ are nonzero vectors then (the cosine of) the angle between $\mathbf{x}$ and $\mathbf{y}$ is equal to (the cosine of) the angle between $T(\mathbf{x})$ and $T(\mathbf{y})$. One can prove that T is conformal if and only if $T = rS$, where $r \neq 0$ and S is orthogonal; you may assume this.

(a) Show that the product of two conformal linear transformations on V is conformal, and the inverse of a conformal linear transformation is also conformal.

(b) Show that a conformal linear transformation is normal.

4. For $k = 0, ..., 10$ determine whether the symmetric matrix

$$\begin{pmatrix} 2 & 4 \\ 4 & k \end{pmatrix}$$

is positive definite, positive semidefinite but not positive definite, or neither.

5. If α and β are the minimum and maximum eigenvalues of a real symmetric matrix A, use diagonalization to show that the Rayleigh quotient

$$\alpha \quad \leq \quad \frac{\langle Ax, x\rangle}{\langle x, x\rangle} \quad \leq \quad \beta$$

for all nonzero vectors x, and that the extreme values are realized for eigenvectors of A.

6. Results from Chapter 10 will show that the symmetric 3×3 matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

is positive definite. Applying the inequalities in the previous exercise with $x = (1, 1, 1)$, find a lower estimate for the largest eigenvalue of A.

Note. One can find an upper estimate for the smallest eigenvalue by doing a similar calculation with A^{-1} replacing A; recall that A^{-1} is symmetric if A is symmetric.