Math 132
Winter 2017

UPDATED GENERAL INFORMATION — MARCH 10, 2017

Office hours for next week

Once again it will be necessary to change office hours. For next week, office hours will be from 10:30 to 11:30 on Tuesday.

The third in-class examination

As previously announced, this will take place on Wednesday, March 8. It will cover material in Chapters 8 and 10 in the text, with modifications as in the online notes, and it will consist of four problems.

Here are a few practice problems, in addition to the assignments already posted, which contain numerous other computational exercises.

1. Axler, Exercise 10A.9.

2. Axler, Exercise 10A.13.

3. Let A be an $n \times n$ matrix over $\mathbb{R}$ such that $A^2 = I$, and let V_+ and V_- be the subspaces spanned by the eigenvectors for 1 and -1 respectively. Prove that V is the direct sum (zero intersection) of V_+ and V_- by considering the images of $\frac{1}{2}(I+T)$ and $\frac{1}{2}(I-T)$. Explain why this implies the identity $\mathrm{trace}(A) = \dim V_+ - \dim V_-$. Prove that

4. Let λ be a nonzero scalar, and let A be the elementary Jordan matrix

$$\begin{pmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & \lambda \end{pmatrix} .$$

Explain why A^3 is not in Jordan form, and find the Jordan form for A^3.

5. Suppose that A is a 4×4 matrix with positive determinant. Explain why the determinant of $-A$ is also positive.

6. Find the determinant of the following matrix:

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 6 & 1 & 0 & 0 \\ 0 & 7 & 8 & 1 & 0 \\ 0 & 9 & 1 & 1 & 0 \\ 5 & 4 & 3 & 2 & 1 \end{pmatrix}$$

7. Find the characteristic polynomials of the following matrices:

$$\begin{pmatrix} 3 & 1 & 0 \\ 0 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix} , \qquad \begin{pmatrix} 2 & 0 & 0 \\ -2 & -2 & 2 \\ 5 & -10 & 7 \end{pmatrix}$$

8. Show that the following real symmetric, positive definite matrix has no rational eigenvalues:

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

You may use without proof the following result due to C. F. Gauss: Suppose that the polynomial

$$a_n z^n \; + \; \cdots \; + a_1 z \; = \; a_0 \qquad (a_n \neq 0)$$

has integer coefficients, and it also has a rational root p/q, where p and q are integers with $q \neq 0$. Then q (evenly) divides a_n and p (evenly) divides a_0. Why does this reduce the search for rational roots to a finite number of possibilities? To conclude, it suffices to verify that none of these rational numbers p/q of this type can be a root of the original polynomial.

9. Suppose that the 4×4 matrix A has two eigenvalues, namely 2 and 3, such that the spaces spanned by eigenvectors for 2 and 3 are both 1-dimensional. Find all possible Jordan forms for A.

10. Suppose that the minimal polynomial of a 4×4 matrix is z^2. Find all possible Jordan forms for A.