Assigned exercises for Chapter 5

Axler 5A: 2–4, 6, 9, 11, 15, 18, 21

Additional exercises

X1. Find the eigenvalues and eigenvectors for the following 2×2 matrices:

$$\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \quad \begin{pmatrix} 0 & -5 \\ 1 & 6 \end{pmatrix} \quad \begin{pmatrix} 8 & 5 \\ -2 & 1 \end{pmatrix}$$

X2. For $\theta \neq n\pi$ $(n \in \mathbb{Z})$ prove algebraically that the rotation matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ has no real eigenvectors.

Axler 5B: 2, 6, 7, 14, 15, 20

X1. An upper triangular $(n \times n)$ matrix is said to be strictly (upper) triangular if its diagonal entries are all zero. Prove that a strictly triangular matrix A is nilpotent, and in fact $A^n = 0$. [Why does A map the subspace W_k spanned by the first k unit vectors into W_{k-1}?]

X2. An upper triangular matrix A is said to be unitriangular if the diagonal entries are each equal to 1. Prove that the product of two $n \times n$ unitriangular matrices is also unitriangular.

Axler 5C: 1-3, 8, 9, 12, 16

X1. Let $p(t)$ be a polynomial, and let A be an upper triangular matrix. Prove that the diagonal entries of $p(A)$ are the scalars $p(a_{ii})$

X2. (A version of the Cayley-Hamilton Theorem) Suppose now that $p(t) = (t - a_{11}) \cdot \dots \cdot (t - a_{nn})$, where A is as above*. Prove that $p(A)$ ~~has zeros down the diagonal~~ $= 0$. [It suffices to verify $p(A)v_j = 0$ where $\{v_1, \dots, v_n\}$ is a basis of eigenvectors].

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* AND $a_{11}, \dots, a_{n,n}$ are distinct.