

Solutions to problems in the file

cab Update 03.132.w17.pdf

1. This matrix is not diagonalizable. The eigen values are given by the diagonal entries and hence are 1 and 2. However

$$A - I = \begin{pmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and the solutions to}$$

$$(A - I) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{are given by all multiples of } \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

The eigen ~~vectors~~ vectors for 2 are given by

$$\begin{pmatrix} -1 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and hence are all multiples of } \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

So there are exactly two linearly indep. eigen ~~values~~ ^{vectors}, one for each eigen value.

(Strictly speaking, the maximum number is 2 and not 3).

2. Solve $\begin{vmatrix} 1-t & -1 \\ 1 & 1-t \end{vmatrix} = 0$

$$t^2 - 2t + 2 = 0. \text{ Then } t = \frac{2 \pm \sqrt{4-4 \cdot 2}}{2} =$$

$$\frac{1}{2}(2 \pm 2i) = 1 \pm i. \text{ The corresponding}$$

eigenvectors are given by the solutions to

$$\begin{pmatrix} -i & -1 \\ 1 & -i \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} i & -1 \\ 1 & i \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

} or all non zero multiples of

$\begin{pmatrix} i \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -i \\ 1 \end{pmatrix}$. Their inner product is

$$(i)(-i) + 1 \cdot 1 = i \cdot i + 1 = i^2 + 1 = 0.$$