

## Assigned exercises for Chapter 10

Axler 10A: 3, 5, 7, 9, 13, 16, 18.

Additional exercises

X1. If  $A$  is an  $n \times n$  matrix over  $\mathbb{C}$  and  $A^*$  is its adjoint, express  $\text{trace } A^*$  in terms of  $\text{trace } A$ .

X2. Show that all matrices of the following types have trace zero:

- (a) Real skew-symmetric matrices.
- (b) Nilpotent matrices.

Axler 10B: 1, 3a, 4-6

Additional exercises

X1. Determine the sign of the permutation  $(1 \dots k)$ . Note The answer depends upon whether  $k$  is even or odd.

X2. Compute

$$\det \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}.$$

X3. If  $A$  is a  $4 \times 4$  skew-symmetric matrix, show that  $\det A = (a_{12}a_{34} + a_{14}a_{23} - a_{13}a_{24})^2$ .

X4. Find the determinants of the following matrices:

$$\begin{bmatrix} 2 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 3 & 2 \\ 1 & 0 & 0 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 3 & 0 \\ 3 & 1 & 3 & 1 \\ 0 & 0 & 2 & 1 \\ 6 & 3 & 4 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

X5. Let  $A$  be an  $m \times n$  matrix. Show that the rank of  $A$  is the largest integer  $q$  such that  $A$  contains a  $q \times q$  submatrix whose determinant is nonzero.

X6. Find the determinants of the following matrices:

$$\begin{bmatrix} 1 & 2 & 13 \\ 4 & 2 & 15 \\ -15 & 2 & 6 \\ 4 & 2 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 & 9 & 12 \\ 1 & 2 & 2 & 1 \\ 3 & 5 & 2 & 1 \\ 0 & 2 & 4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{bmatrix}$$