Solutions to problems in the file

aab Update 03.132.w17.pdf

1. This matrix is not diagonalizable. The eigen values are given by the diagonal entries and hence are 1 and 2. However

$$A - I = \begin{pmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ and the solutions to

$$(A-I)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$ are given by all multiples of $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.

The eigen ~~values~~ vectors for 2 are given by

$$\begin{pmatrix} -1 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$ and hence are all multiples of $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

So there are exactly two linearly indep. eigen~~values~~vectors, one for each eigen value.

(Strictly speaking, the maximum number is 2 and not 3).

2. Solve $\begin{vmatrix} 1-t & -1 \\ 1 & 1-t \end{vmatrix} = 0$

$t^2-2t+2=0$. Then $t = \frac{2 \pm \sqrt{4-4 \cdot 2}}{2} =$

$\frac{1}{2}(2 \pm 2i) = 1 \pm i$. The corresponding eigenvectors are given by the solutions to

$$\left.\begin{array}{l} \begin{pmatrix} -i & -1 \\ 1 & -i \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} i & -1 \\ 1 & i \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \end{array}\right\} \text{or all nonzero multiples of}$$

$\begin{pmatrix} i \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -i \\ 1 \end{pmatrix}$. Their inner product is

$(i)\overline{(-i)} + 1 \cdot 1 = i \cdot i + 1 = i^2 + 1 = 0.$