NAME: ______________________________

Mathematics 132, Winter 2021, Examination 2

INSTRUCTIONS: Work all questions, and unless indicated otherwise give reasons for your answers. The point values for individual problems are indicated in brackets. Please submit a copy of your exam by electronic mail, one to **each** of the following addresses, by **11:59 P.M.** on **Wednesday, March 17, 2021:**

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Each problem should be started on a separate sheet of paper. Make sure your submission is readable; some smartphone photos might not have enough contrast. You may look at outside references such as course directory documents before starting this examination, and you may consult with other students, the teaching assistant or me about material related to this examination, but this assignment is **NOT** collaborative. The answers you submit must be your own work and nobody else's; this includes use of mechanical or electronic computational devices.

The top score for setting the curve will be 150 points.

Please make sure that an electronic copy of your completed exam is also sent to your email account in case there are unexpected transmission problems.

1. [*25 points*] (a) Let A be a real symmetic $n \times n$ matrix with real entries that is positive definite. Prove that there is a minimum value $c^* > 0$ such that $A - c^* I$ is **NOT** positive definite. [*Hint:* Look first at the case where A is diagonal. Why does this shed light on the general case?]

(b) Find c^* when A is the following matrix:

$$\begin{pmatrix} 8 & 3 \\ 3 & 5 \end{pmatrix}$$

2. [*25 points*] Suppose that A is a 3×3 orthogonal matrix, and consider the normal form as described in `week06/orthog-nform.pdf`. Show that this matrix must have a 2×2 block summand if $A^2 \neq I$. (There is also a converse: If $A^2 = I$ then the normal form is a block sum of 1×1 matrices.)

3. [*25 points*] Let $\lambda \neq 0$ be a scalar, and let N be the matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

so that $\lambda I + N$ is an elementary Jordan matrix. Show that $(\lambda I + N)^3$ is not in Jordan form, and show that its Jordan form is $\lambda^3 I + N$. [*Hint:* What is $(\lambda I + N)^3 - (\lambda^3 I)$?]

4. [*25 points*] (a) Find the eigenvalues for the following real symmetric matrix:

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 4 \end{pmatrix}$$

(b) For each rational eigenvalue α, find a basis for the space of associated eigenvectors.

5. [*25 points*] Find the determinant of the matrix displayed below. You may use any valid method to carry out the computation(s).

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 3 & 4 \\ 2 & 2 & 3 & 4 \\ 3 & 3 & 3 & 4 \end{pmatrix}$$

6. [*25 points*] Let V be an n-dimensional inner product space over the complex numbers, suppose that $T : V \to V$ is a normal operator, and let $n \geq 2$ be an integer. Prove that T^n is also normal.