EXERCISES FOR CHAPTER 6

6A. Inner products

Exercises in Axler: 2, 4, 8, 10, 11, 12, 16, 20, 23, 27, 31

Additional exercises:

X1. Find a nonzero vector in $\mathbb{R}^4$ which is perpendicular to each of $(1,1,1,0)$, $(0,1,1,1)$ and $(1,0,0,1)$. [This amounts to solving a system of three homogeneous linear equations in four unknowns.]

X2. Find a pair of linearly independent vectors in $\mathbb{R}^4$ which are perpendicular to both $(1,2,3,4)$ and $(5,6,7,8)$.

X3. There is a very simple realization of a regular tetrahedron (= pyramid with triangular base) in $\mathbb{R}^4$ such that the vertices are the usual four unit vectors $\mathbf{e}_i$, where $i = 1,2,3,4$ and the center $\mathbf{b}$ is their average value $\frac{1}{4}\,\Sigma\,\mathbf{e}_i$. Find the cosine of the angle whose edges are $\mathbf{e}_1 - \mathbf{b}$ and $\mathbf{e}_2 - \mathbf{b}$, and find the degree measure of this angle. The drawing in `exercises6Afigure.pdf` might be helpful.

6B. Orthogonality and dimension

Exercises in Axler: 1, 2, 3, 10, 16

Additional exercises:

X1. (a) Express the vector $(2,2,3) \in \mathbb{R}^3$ as a sum $x + y$ where x is a multiple of $(1,1,0)$ and y is perpendicular to $(1,1,0)$.

(b) Express the vector $(1,-1,1) \in \mathbb{R}^3$ as a sum $x + y$ where x is a multiple of $(1,1,1)$ and y is perpendicular to $(1,1,1)$.

X2. Find orthogonal bases for the spans of the following vector triples in $\mathbb{R}^4$:

(a) $(0,0,1,1)$, $(0,1,1,0)$, $(0,0,1,1)$

(b) $(1,1,1,1)$, $(-1,4,4,1)$, $(4,-2,-2,0)$

6C. Orthogonal complements, projections, least squares

Exercises in Axler: 3, 4, 5, 10, 11

Additional exercises:

X1. Suppose that U and W are subspaces of the finite dimensional inner product space V. Prove that one has the identity $U^\perp \cap V^\perp \;=\; (U+V)^\perp$.

X2. Let W be a subspace of the finite dimensional inner product space V, and let E denote the linear transformation $V \to V$ given by orthogonal projection onto W. Prove that $T = (1 - 2E)$ satisfies $T^2 = 1$. Also, show that every nonzero vector in W is an eigenvector for T.

X3. Let $W \subset \mathbb{R}^3$ be the subspace spanned by $(1,1,0)$ and $(0,1,1)$. If E is orthogonal projection onto W, find the matrix of E with respect to the standard unit vector basis; in other words, evaluate $E\mathbf{e}_i$ where $i = 1,2,3$.