NAME: ____________________________

Mathematics 132, Winter 2021, Examination 1

INSTRUCTIONS: Work all questions, and unless indicated otherwise give reasons for your answers. The point values for individual problems are indicated in brackets. Please submit a copy of your exam by electronic mail, one to **each** of the following addresses, by 11:59 P.M. on Tuesday, February 16, 2020:

`rschultz@ucr.edu`   `jwagn006@ucr.edu`

Each problem should be started on a separate sheet of paper. Make sure your submission is readable; some smartphone photos might not have enough contrast. You may look at outside references such as course directory documents before starting this examination, and you may consult with other students, the teaching assistant or me about material related to this examination, but this assignment is **NOT** collaborative. The answers you submit must be your own work and nobody else's.

The top score for setting the curve will be 100 points.

#	**SCORE**
1	
2	
3	
4	
TOTAL	

1. [*25 points*] Suppose that we have an ordered orthonormal basis $\mathbf{B} = \{x_1, ..., x_n\}$ for an inner product space V. Let $\{u_1, \cdots, u_n\}$ be the ordered orthonormal basis obtained from $\mathbf{B}$ by the Gram-Schmidt orthonormalization proces. Prove by induction that $u_k = x_k$ for $k = 1, \cdots, n$.

2. [*25 points*] Let A, B, C and D be the last four digits of you student identification number in that order. Fid the least squares approximation for the following data:

x =	1	2	3	4	5
y =	1+0.A	2–0.B	3	4+0.C	5–0.D

See the document `.../week02/least-squares-example.pdf` for a solved problem of this type.

3. [*25 points*] Given a complex number $z = a + bi$, show that the map M sending z to

$$M(z) \quad = \quad \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

is normal and satisfies $M(z+w) = M(z) + M(w)$ and $M(zw) = M(z)M(w)$. Also show that the eigenvalues are $a \pm bi$.

4. [*25 points*] Let A be a square matrix over the complex numbers. Prove that each of $A + A^*$, $A - A^*$, and AA^* has an orthonormal basis of eigenvectors, and the eigenvalues are real in first and last cases and imaginary in the second.