

More on Axler 7C9.

Verification that $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}^2 = \mathbf{I}$.

Let $\alpha = 2\theta$ for simplicity. Then

$$\begin{pmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & -\cos \alpha \end{pmatrix} \begin{pmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & -\cos \alpha \end{pmatrix} =$$

$$\begin{pmatrix} \cos^2 \alpha + \sin^2 \alpha & \cos \alpha \sin \alpha - \sin \alpha \cos \alpha \\ \cos \alpha \sin \alpha + \sin \alpha \cos \alpha & \sin^2 \alpha + \cos^2 \alpha \end{pmatrix} =$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$