SOLUTIONS TO PROBLEMS IN AXLER, SECTION 10B

The solutions to all problems except number 6 are in **http://linearalgebras.com/10b.html**. Here is the solution for **6:**

For each matrix A_k, we know that A_k is similar to an upper triangular matrix C_k, where the latter is the matrix of the associated linear transformation "A_k" with respect to the ordered basis $\mathcal{B}_k$. If we form an ordered basis $\mathcal{B}$ from the latter by taking the vectors in $\mathcal{B}_1$ first, $\mathcal{B}_2$ second, and so on, then the matrix C of the linear transformation "A" with respect to the ordered basis $\mathcal{B}$ will again be upper triangular, for each basis vector in $\mathcal{B}$ is sent to a linear combination of itself and the preceding vectors. Furthermore, the resulting diagonal block submatrices C_k must also be upper triangular for the same reason.

The determinants of C and A are equal because the matrices A and C are similar because similar matrices have the same determiinants, and likewise for the submatrices A_k and C_k. Now the diagonal entries of C are given by the corresponding diagonal entries for all of the C_k's, and therefore the determinant of C equals the product of the determinants of the submatrices C_k. Since C is similar to A and each submatrix C_k is similar to the corresponding submatrix A_k, the invariance of determinants with respect to similarity implies that the determinants of C and each submatrix C_k are equal to the determinants of A and each A_k respectively, and hence $\det A = \det C$. Since the latter is the product of all the scalars $\det C_k$, it follows that the former is equal to the product of the (corresponding equal) scalars $\det A_k$. ■