Math 132
Spring 2018

**UPDATED GENERAL INFORMATION — MAY 3, 2018**

*The midterm examination*

This examination will consist of four problems (some with a few parts). About 60 per cent will involve computations and definitions, and the remaining 40 per cent will involve simple proofs or derivations, with allowances for partial credit. Material up to and including Section 7A will be covered.

Homework and exercise files in the course directory provide good material for review and preparation, and so do examinations from last year. A new file of problems for Section 7A will be posted this afternoon. It will also be worthwhile to be aware of some key examples, like matrices which cannot be diagonalized over either the real or complex numbers and why this is the case. Here is one example to consider:

$$\begin{pmatrix} 1 & 3 & 5 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{pmatrix}$$

Here is another problem:

**1.** Find the eigenvalues and eigenvectors for the matrix

$$\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix} .$$

We should note that there are no real eigenvalues in this case.

And here are some more problems that are worth considering:

**2.** Suppose that $v$ and $w$ are vectors in a real inner product space $V$ such that $|v+w| = 12$ and $|v-w| = 13$. Use the polarization identity to compute $\langle v, w \rangle$.

**3.** Let $A$ be a $2 \times 2$ matrix, and let $A^{\mathbf{T}}$ denote its transpose. Show that these two matrices have the same eigenvalues, and give an example where there are two real eigenvalues but the associated eigenvectors for the two matrices are distinct (if $\lambda$ is the eigenvalue and $p, q$ are the associated eigenvectors for $A$ and $A^{\mathbf{T}}$, then neither $p$ nor $q$ is a multiple of the other).

**4.** Let $V$ be a finite dimensional inner product space over the real or complex numbers, and let $T : V \to V$ be a self adjoint linear transformation. Given two **integers** $a$ and $b$, prove that $T^3 + aT^2 + bT$ is self adjoint. Explain why the conclusion does not hold if $a$ and $b$ are arbitrary complex numbers.

**5.** Let $V$ be a finite dimensional inner product space over the complex numbers, and let $T : V \to V$ be a linear transformation. Prove that $i(T - T^*)$ is self adjoint.