Math 132
Spring 2018

UPDATED GENERAL INFORMATION — MAY 21, 2018

The third quiz

The third quiz, which is scheduled for Thursday, May 3, will cover material from Sections 7A, 7B, 7C and 8A. As usual it is worthwhile to go through the exercise and solution files as well as old exams. Here are some further problems to study in connection with the quiz.

1. Using only the definitions (*i.e.*, not the Spectral Theorem or its converse), find all values of t such that the matrix

$$\begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$$

is normal.

2. Suppose that $a > 0$. Find all t such that the real symmetric matrix

$$\begin{pmatrix} 4a & a \\ a & t \end{pmatrix}$$

is positive definite.

3. Suppose that N is a nilpotent matrix whose index of nilpotency is p. For each $k > 0$, prove that N^k is nilpotent and that its index of nilpotency is less than or equal to $(p+k)/k$.

4. Suppose that N is a nilpotent matrix such that $N^3 = 0$. Prove that there is a matrix $S = I + B$ such that $S^2 = I + N$. [*Hint:* If we substitute N into the infinite series expansion for $\sqrt{1+x}$

$$(1+x)^a \quad = \quad \sum_{k=0}^{\infty} \binom{a}{k} x^k \ , \quad \text{where} \quad \binom{a}{k} \quad = \quad \frac{a \cdots (a-k+1)}{k!} \quad \text{if} \quad k > 0$$

(and $a = \frac{1}{2}$) then only finitely many terms are nonzero.]