Math 132
Spring 2018

UPDATED GENERAL INFORMATION — JUNE 4, 2018

Practice problems for the second exam

Here are a few beyond the usual sources:

1. Let V be a finite dimensional inner product space, let $T : V \to V$ be a normal linear transformation, and let a, b, c be scalars. Prove that $aT^2 + bT + cI$ is also normal.

2. Suppose that A is a symmetric 2×2 matrix over the reals such that $\det A > 0$ and $\operatorname{tr} A > 0$. Explain why A has only positive real eigenvalues. Give examples to show this fails for 3×3 matrices which are symmetric and 2×2 matrices which are not symmetric.

3. Find all Jordan forms for 6×6 matrices with minimal polynomial $(t^2 - 4)^2$.

4. A linear transformation $T : V \to V$ on a vector space is said to be an **involution** if $T^2 = I$. Since our scalars are real or complex numbers, it follows that V is a direct sum of the eigenspaces $V_\pm$ of ± 1. If V a finite dimensional inner product space, show that T is self adjoint if and only if these two eigenspaces are orthogonal complements of each other.

5. Find the determinant of the following 4×4 matrix:

$$\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

6. Prove that a symmetric nilpotent matrix over the real numbers must be the zero matrix.

7. Find the eigenvalues of the following 3×3 matrix:

$$\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

8. Let A be a real 2×2 matrix. Prove that A does not have a basis of eigenvectors over either the real or complex numbers if and only if A is not a multiple of the identity matrix and $(\operatorname{tr} A)^2 = 4 \det A$.