

Situation

$\{w_1, \dots, w_k\}$ nonzero, mutually \perp

v not in $\text{Span}\{w_1, \dots, w_k\}$

Construction

$$w^* = v - \sum_{j=1}^k \frac{\langle v, w_j \rangle}{\langle w_j, w_j \rangle} w_j.$$

Second condition implies $w \neq 0$.

Claim $w^* \perp w_j$ all j .

Verification For all i $\langle w^*, w_i \rangle =$

$$\langle v, w_i \rangle - \sum_{j=1}^k \frac{\langle v, w_j \rangle}{\langle w_j, w_j \rangle} \langle w_j, w_i \rangle =$$

0 if $i \neq j$

$$\langle v, w_i \rangle - \frac{\langle v, w_i \rangle}{\langle w_i, w_i \rangle} \langle w_i, w_i \rangle = 0.$$

This holds for all $i = 1, \dots, k$.