EXAMPLE OF A LEAST SQUARES PROBLEM

Find the least squares line for the data

x =	1	2	3	4	5
y =	2	5	3	8	7

Let X and Y be the 5×1 column vectors for these values. We want to minimize

$$|Y - (aX + bU)|^2$$

where the entries of U are all ones. The displayed expression is $\sum (y_i - (ax_i + b))^2$.

To apply linear algebra, we must replace X and U by an orthogonal set spanning the same 2-dimensional subspace of $\mathbb{R}^5$. If $\bar{x}$ is the average x-value $\frac{1}{5}\sum x_i$, then this is given by $\overline{X} = X - \bar{x}U$ and U; then $\overline{X} \cdot U = 0$ and $\{\overline{X}, U\}$ is linearly independent since the coords of X are distinct. We then have

$\boxed{\bar{x} = 3 \text{ here}}$

$$|Y - (aX + bU)|^2 = |Y - (a\overline{X} + (b + a\bar{x})U)|^2$$

which is a form we saw in Axler, Chapter 6.

We then know the minimum occurs when

$$a = \frac{\langle Y, \overline{X} \rangle}{\langle \overline{X}, \overline{X} \rangle} = \frac{13}{10}$$

$$b + a\bar{x} = \frac{\langle Y, U \rangle}{\langle U, U \rangle} = \frac{25}{5} = 5$$

The left hand side is $b + \frac{13}{10} \cdot 3 = b + \frac{39}{10}$

and hence $b + \frac{39}{10} = 5$, so that $b = \frac{11}{10}$.

Hence the least squares line is

$$\boxed{y \approx 1.3x + 1.1}$$ ∎