

Derivations of ① and ②

① $x = \sum c_j w_j$, w_j orthonormal. \Rightarrow

$$\langle x, w_i \rangle = \sum c_j \langle w_j, w_i \rangle = c_i \cdot 1 = c_i.$$

*0 if $j \neq i$
1 if $j = i$*

② $\langle x, y \rangle = \langle \sum_i a_i w_i, \sum_j b_j w_j \rangle =$

$$\sum_{i,j} a_i \bar{b}_j \langle w_i, w_j \rangle = \sum_i a_i \bar{b}_i$$

since $\langle w_i, w_j \rangle = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$