Triangular matrices

Here are two examples of triangular matrices. The first is upper triangular, and the second is lower triangular.

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 3 & 3 \end{pmatrix}$$

Two types of triangular matrices have special names. A triangular matrix is **strictly triangular** if all the diagonal entries are zeros, and it is **unitriangular** if these entries are all ones. Many writers also use the word **unipotent** to describe the latter class of matrices.

If a matrix B is upper triangular, then the linear transformation sending a column vector X to BX has the following important property: *If E_j denotes the unit column vector whose j^{th} coordinate is 1 and all other coordinates are 0, then BE_j is a linear combination of the first j column vectors.* — Conversely, if B has the latter property, then it is upper triangular.

More generally, if $T : V \to V$ is a linear transformation and $\mathcal{B}$ is an ordered basis $\{b_1, \cdots, b_n\}$ for V, we say that T is in upper triangular form with respect to $\mathcal{B}$ if for each j the vector Tb_j is a linear combination of $b_1, \cdots, b_j$.