Example of the Power Method for finding the maximal eigenvalues and their eigenvectors

The Power Method proceeds as follows: We start with a square matrix **A** and a nonzero column vector **x**, say the first unit vector, such that the first coordinate of **Ax** is nonzero. Our example is the following positive definite symmetric matrix which we have already studied:

1	1	1
1	2	2
1	2	3

We proceed to consider the sequences of vectors $\{\mathbf{x}_n\}$ and $\{\mathbf{y}_n\}$ defined by $\mathbf{x}_1 = \mathbf{e}_1$ and recursively $\mathbf{y}_{n+1} = \mathbf{A}\mathbf{x}_n$ and $\mathbf{x}_{n+1} = \mathbf{d}_{n+1}^{-1}\mathbf{y}_{n+1}$, where $\mathbf{d}_{n+1}$ is the first coordinate of $\mathbf{y}_{n+1}$. If this coordinate is **0** at some point, then the procedure does not work, but it does work most of the time. If we iterate this process enough times we see that eventually the vectors $\mathbf{x}_n$, $\mathbf{y}_n$ and the scalar dn seem to approach a limit; in practical terms, the approximations up to **N** decimal places (where **N** is arbitrary) eventually reach a stable value. The limit of the values $\mathbf{d}_n$ turns out to be the largest eigenvalue of the matrix, and the limits of the vectors $\mathbf{x}_n$ and $\mathbf{y}_n$ are eigenvectors for this eigenvalue. A proof of these facts is given in a companion document; the table below shows what happens for our example. One can read off the eigenvalue approximations $\mathbf{d}_n$ as the entries in the fourth column (= the first column in the middle group).

	$\mathbf{x}_n$				$\mathbf{y}_{n+1} = \mathbf{A}\mathbf{x}_n$				$\mathbf{x}_{n+1}$	
1	0	0		1	1	1		1	1	1
1	1	1		3	5	6		1	1.666667	2
1	1.666667	2		4.666667	8.333333	10.33333		1	1.785714	2.214286
1	1.785714	2.214286		5	9	11.21429		1	1.8	2.242857
1	1.8	2.242857		5.042857	9.085714	11.32857		1	1.8017	2.246459
1	1.8017	2.246459		5.048159	9.096317	11.34278		1	1.801908	2.246914
1	1.801908	2.246914		5.048822	9.097643	11.34456		1	1.801934	2.246971
1	1.801934	2.246971		5.048905	9.09781	11.34478		1	1.801937	2.246979
1	1.801937	2.246979		5.048916	9.097832	11.34481		1	1.801938	2.246979
1	1.801938	2.246979		5.048917	9.097834	11.34481		1	1.801938	2.24698
1	1.801938	2.24698		5.048917	9.097835	11.34481		1	1.801938	2.24698
1	1.801938	2.24698		5.048917	9.097835	11.34481		1	1.801938	2.24698
1	1.801938	2.24698		5.048917	9.097835	11.34481		1	1.801938	2.24698
1	1.801938	2.24698		5.048917	9.097835	11.34481		1	1.801938	2.24698
1	1.801938	2.24698		5.048917	9.097835	11.34481		1	1.801938	2.24698