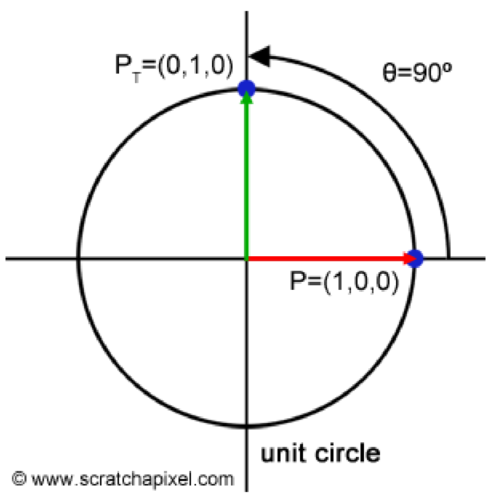
A matrix with no real eigenvalues

The standard geometric interpretation of the linear transformation with matrix

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

is that it represents a counterclockwise rotation by 90 degrees about the origin. From this viewpoint it is clear that the matrix has no real eigenvalues or eigenvectors, for it takes a vector (x, y) to its image under this rotation. The rotated vector is never a scalar multiple of the original vector provided the latter is nonzero, and hence there cannot be any real eigenvectors or eigenvalues.



The Spectral Theorem imples that the matrix has an orthonormal basis of eigenvectors over the complex numbers but has no eigenvectors over the reals. A similar conclusion holds for all 2x2 rotation matrices except for the angles of 0 and 180 degrees (when A = -I).